

Effect of Stacking Sequence Variation on First Ply Failure Load of Laminated Composite Skew Plate

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Abstract

The present work aims to study and report the first ply failure load of laminated composite skew plate of different laminations and stacking sequences with a practical boundary condition. The finite element model is developed using eight noded isoparametric curved quadratic shell element with five degrees of freedom per node (with three translations and two rotations). Numerical experiments are conducted considering skew plates of varying skew angles, laminations and stacking patterns. First ply failure loads are evaluated for seven different failure criteria such as maximum stress, maximum strain, Tsai-Wu, Tsai-Hill, Hoffman, Puck and Hashin's failure criteria. The minimum value of load obtained from different failure criteria is chosen as the governing first ply failure load value. Benchmark problems are solved to validate the correctness of the present approach of evaluation of first ply failure load and incorporation of skew geometry in the present model. The results of authors' own problems are finally studied meticulously and relative performance matrix of skew plate options of different combination of skew angle and stacking sequences are reported.

Keywords: First ply failure, laminated composite, skew plate, stacking pattern, failure mode, failure criteria

1. Introduction

The practice of design and engineering has always intended to explore material which offers high strength, stiffness, flexibility and reliability over existing natural and man-made construction material. This desire has opened a wide area of research on the application of laminated composite materials on different branches of civil and structural engineering. Among the different structural elements, slab or plate element is one of the significant components in any civil engineering structure. Further in urban areas where there is constraint of enough space conventional rectangular and symmetrical buildings cannot be built effortlessly. The limited and irregular space may demand slab panels to be shaped other than normal rectangular or square geometry.

Use of laminated composite material in construction of roof or floor slab calls for a detailed study of response of such material under static transverse uniformly distributed load. One such response is the first ply failure and further progressive failure of laminated composite plates. In the last three decades many researchers were involved in detailed study of failure behaviour of such laminated composite plates and shell roofs of various geometries with different practical parametric variations. Recently Biswas and Ray [1] studied the impact of hybridisation on the strength of laminated composite fully clamped plate under transverse static load. A comparison was done between the failure strength of carbon and glass hybridised fabric laminate with

epoxy resin (HFRP) and glass fabric –epoxy laminate (GFRP). Authors observed that there was marked improvement in first ply failure load values both experimentally and theoretically. Also hybridized (HFRP) panels showed greater velocity of sound wave transmission compared to glass fibres (GFRP) laminate. Chakrabarty et al. [2] investigated the governing failure modes of first ply failure loads of laminated composite sandwich plates using improved higher order shear deformation theory (IHSDT). Different boundary conditions were considered beside laminations, ply orientation, aspect ratios, thickness and load values for uniformly distributed load and sinusoidal loading. It was found that for almost all parametric variations the governing failure mode remains either matrix cracking or fibre breakage or inter fibre shear failure of the matrix.

Laxminarayana et al. [3] reported the progressive failure of laminated composite plate with elliptical and circular cutouts and without cutout using Hashin's failure criterion. The effect of cut out size, orientation, plate thickness on ultimate failure strength was taken in to consideration. The plates were subjected to uniaxial compressive load. Chakrabarty et al. [4] proposed for the first time first ply failure load analysis of laminated composite skew plate geometry with skew angles varying from 0° to 45° at an interval of 15° under uniformly distributed transverse load. Authors considered 9 noded

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isoparametric element with seven degrees of freedom per node with C^0 element for finite element formulation using higher order shear deformation theory. Both angle ply and cross ply laminates were studied under fully simply supported and clamped boundary conditions. Failure load of angle ply laminated curved panels subjected to compressive load for simply supported and clamped boundary conditions were studied and reported by Adali et al. [5]. Failure load considered was the minimum among the buckling and first ply failure load. Effect of fibre orientation on failure load with different boundary condition, aspect ratio and panel thickness were also investigated.

Bending behaviour of laminated composite skew plates for different types of loadings and support conditions were also scrutinized by Ikharrazne [6]. Skew angles were taken over a wide range of 30° to 90° at an interval of 5° and 10° . Central deflections of simply supported glass-epoxy and graphite epoxy were computed for different mesh sizes and results were cross checked analytically. Bending behaviour of clamped rhombus shaped isotropic plate under uniformly distributed load was investigated by Morley [7]. Bending moments and deflection both at centre and edges were reported for skew angle varying from 22° to 40° at an interval of 2.5° and 5° . Effect of both uniformly distributed as well as concentrated load was studied.

The above review over last four decades reveals that papers on first ply failure behaviour on skew plate configuration was very scanty. Only one paper by

Chakrabarty et al. [2] reported such analogous study. But that too not considered detail effects of varying fibre stacking and ply orientation and number of layers. In the present paper authors make an effort to examine the consequence of variable stacking pattern on first ply failure load of skew plate. Effect of repetitions of angle and cross ply units is also considered. Two new failure criteria as proposed by Hashin and Puck are additionally taken for computation of first ply failure load.

2. Mathematical Formulation

In the present paper authors intended to find the variation of first ply failure load of laminated composite skew plate with the change of ply orientation and stacking pattern.

Fig. 1(a) introduces the laminated composite skew plate geometry of skew angle ' α ' with five degrees of freedom ($u, v, w, \alpha x$ and αy). It consists of planar dimensions of length ' l ' and width ' b ' and uniform thickness ' h '. The plate consists of certain number of lamina along z direction as represented in Fig. 1(a) oriented at angle of ' φ ' with respect to x axis of the plate. The plate is fully clamped along the four edges OA, AB, BC and OC (refer Fig. 1(b)) and subjected to uniformly distributed load. An 8×8 finite element discretization is considered for the present study.

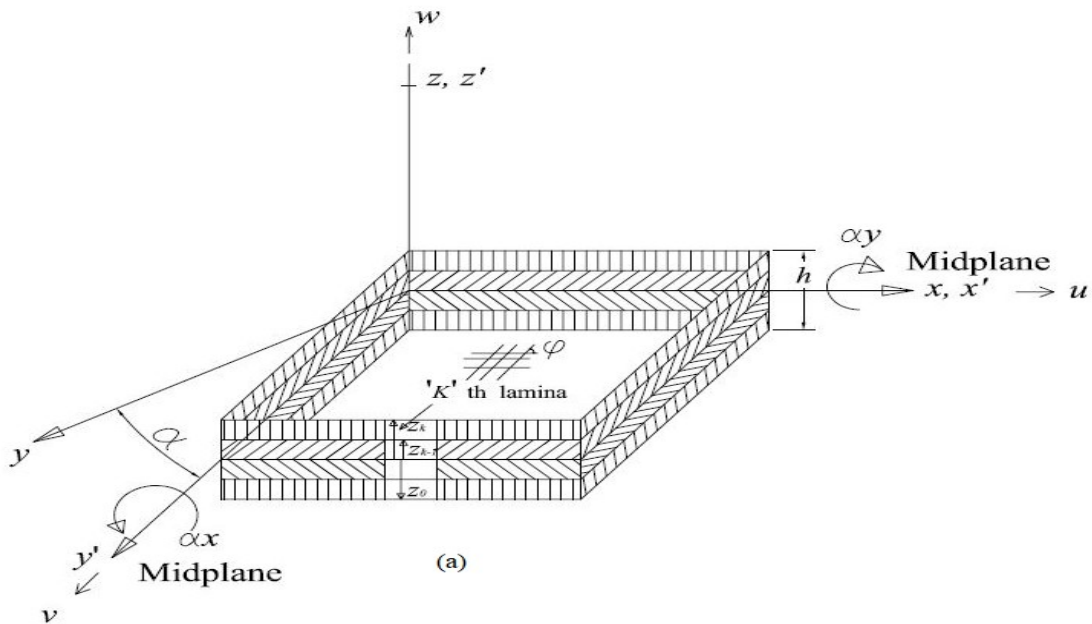


Fig. 1(a). Laminated composite skew plate showing skew angle, fibre orientation and five degrees of freedom

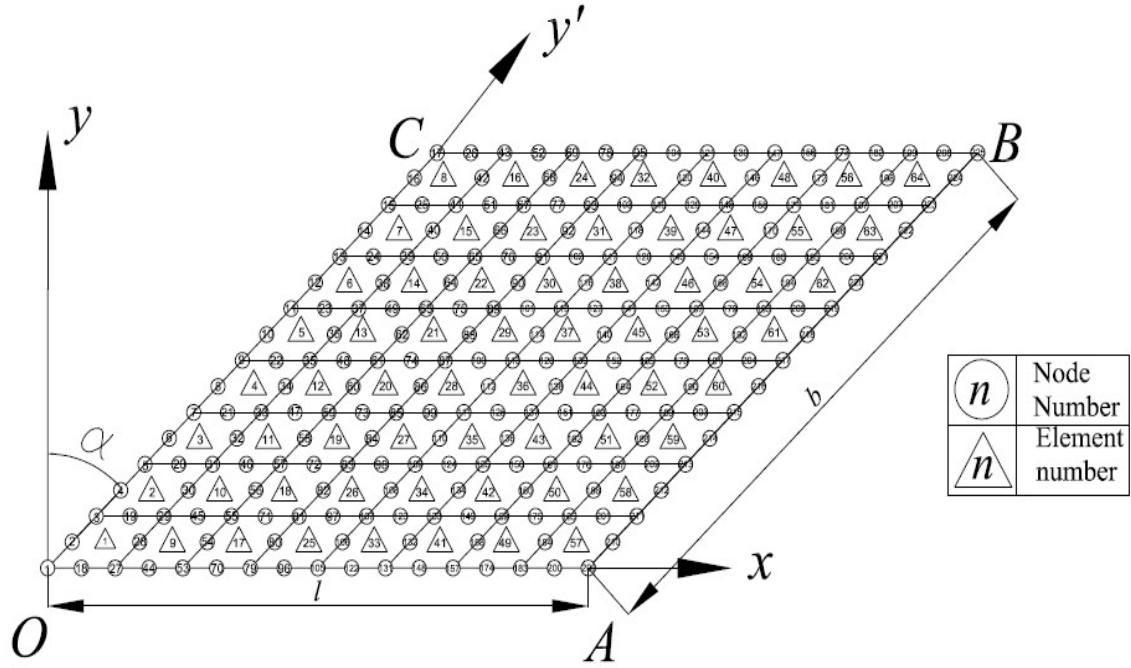


Fig. 1(b). Plan view showing planar dimensions

2.1 Governing equation of plate bending

The plate bending equation is derived from the principle of minimum total potential energy " PE " which is the sum of strain energy " U " and work done by external load " W_e ". The steps describing the minimization of total potential energy, solution of governing differential equation and lamina stress calculation and transformation of lamina stresses and strains from global to local axis are done as per Bakshi and Chakravorty [8].

First ply failure loads are evaluated as per Kam et al. [9] after incorporating the lamina stresses and strains in various failure theories such as maximum stress, maximum strain, Hoffman, Tsai-Hill, Tsai-Wu, Hashin and Puck. The minimum value of failure load among all seven criteria is reported as the first ply failure load which is only reported in this paper for each skew angle.

3. Numerical Study

The correctness of the present approach in predicting first ply failure load of laminated composite skew plate geometry is established through two benchmark problems. Table 1 compares the non dimensional frequency of laminated composite skew plate geometry obtained from the present finite element code with those obtained by Singha et al. [10]. Since no papers are available in the literature to predict first ply failure load values of thin laminated composite skew plate geometry. So Table 2 compares the linear first ply failure load obtained by the present code with those reported by Kam et al. [9] for normal rectangular plates with skew angle 0° .

Table-1. Non-dimensional frequency $\left(\bar{\omega} = \omega a^2 / \pi^2 \sqrt{\frac{\rho}{E_{22}}}\right)$ of five layered $[90^\circ/0^\circ/90^\circ/0^\circ/90^\circ]$ simply supported skew laminates $(l/b = 1)$

Skew angle	Mesh size	Modes	1	2	3
0°	Present study		1.967	4.022	6.688
	Singha and Daripa [10]	8×8	1.914	3.973	6.645
30°	Present study		2.907	5.259	8.497
	Singha and Daripa [10]	8×8	2.838	5.195	8.464

Note- $E_{11}/E_{22} = 40.0$, $G_{12}/E_{22} = 0.6$, $G_{23}/E_{22} = 0.5$, $\nu_{12} = 0.25$, $l/t = 1000$

Table-2. First ply failure loads in Newton for $[0^\circ/90^\circ]_s$ plate

Failure criteria	Side/thickness ratio	Experimental failure loads Kam et al. [9]	First ply failure loads Kam et al. [9]	First ply failure loads (present formulation)
Maximum stress			64.94	63.93
Maximum strain			76.04	92.90
Hoffman	152.67	158.08	63.60	58.02
Tsai-Hill			64.03	72.48
Tsai-Wu			68.30	64.26

Note: Length = 100 mm, ply thickness = 0.155 mm, load details = central point load

Further, numerical experiments are conducted by authors considering fully clamped skew plate configurations of various lamina stacking sequences with 8×8 finite element grid size. The material properties and the

geometrical properties that are used by authors to carry out the numerical experiments are presented in Table 3 and Table 4 respectively. The results of other numerical examples which are carried out by the authors with different parametric variations are furnished in Table 5 and Table 6 and Fig. 2 to Fig. 5. These results are meticulously interpreted from practical engineering point of view.

Table-3. Material properties of Q-1115 graphite-epoxy composite

E_{11}	142.50	GPa	ν_{12}	0.01539
E_{22}	9.79	GPa	ϵ_{1C}	0.01724
E_{33}	9.79	GPa	$\epsilon_{2T} = \epsilon_{3T}$	0.00412
$G_{12} = G_{21}$	4.72	GPa	$\epsilon_{2C} = \epsilon_{3C}$	0.02112
G_{23}	1.192	GPa	γ_{12u}	0.05141
f_{1T}	2193.5	MPa	γ_{23u}	0.01669
f_{1C}	2457.0	MPa	τ_{13u}	0.01669
$f_{2T} = f_{3T}$	41.3	MPa	$\nu_{12} = \nu_{13}$	0.27
$f_{2C} = f_{3C}$	206.8	MPa	ν_{23}	0.25
τ_{12u}	61.28	MPa		
τ_{23u}	78.78	MPa		
τ_{13u}	78.78	MPa		

Table-4. Geometrical dimensions of the skew plate

Skew plate dimensions	Values
Aspect ratio (l/b)	1
Width to thickness ratio (b/t)	100
Skew angles, α	0°, 15°, 30°, 45°

4. Results and Discussions

The close match of the results shown in Table 1 establishes the proper incorporation of the skew plate

geometry in the present computer code. Besides, the first ply failure load values of laminated composite plate from present analysis show very good agreement with the failure load values reported by Kam et al. [9] in Table 2. This proves the correctness of the first ply failure analysis of laminated composite skew plate from present computer code.

Tables 5 and 6 represent the uniformly distributed first ply failure loads of laminated composite fully clamped skew plates of varying skew angles. Seven different failure theories are applied to evaluate the failure loads and the minimum value obtained among the different failure criteria are reported as the first ply failure load.

For cross ply skew plates Hoffman, Puck and maximum strain failure criteria play the governing roles in determination of first ply failure loads. So other failure criteria can be ignored during computation of first ply failure load of clamped cross ply skew plate. It is very practical that for clamped boundary condition maximum stress will occur at support top for transverse loading. So in most of the cases one can find the failure at supports or at elements near support. Here, the results shown in Table 5 and Table 6 also agree to this fact.

In case of clamped angle ply skew plate, the governing failure mode or tendency observed is transverse matrix cracking. As the fibres are not along the global axes of the plate thus failure is mainly due to matrix cracking under tensile stress. For normal rectangular cross ply plate, $[0^\circ/90^\circ/0^\circ]$ gives the greatest first ply failure strength followed by $[0^\circ/90^\circ/0^\circ/90^\circ]$ and $[0^\circ/90^\circ/90^\circ/0^\circ]$. For normal rectangular angle ply clamped plate symmetric four layered laminate $[45^\circ/-45^\circ/-45^\circ/45^\circ]$ shows the best performance in terms of first ply failure load values.

As skew angle increases from 15° to 30° and 45°, two layered antisymmetric clamped cross ply laminate such as $[0^\circ/90^\circ]$ shows better performance than three layered antisymmetric laminate $[0^\circ/90^\circ/0^\circ]$ in general. In case of

Table-5. Uniformly distributed first ply failure load (FL) of cross ply skew plate, location of failure and first failed ply number (K)

Skew angle, α (degree)	Stacking sequence	Governing failure criterion	FL (MPa)	K	Failure node	Failure tendency	mode/failure
0	$[0^\circ/90^\circ]$	Hoffman	0.1050	1	9	Fibre breakage	
	$[0^\circ/90^\circ/0^\circ]$	Maximum strain	0.1657	1	121	Transverse matrix cracking	
	$[0^\circ/90^\circ/0^\circ/90^\circ]$	Maximum strain	0.1538	1	105	Transverse matrix cracking	
	$[0^\circ/90^\circ/90^\circ/0^\circ]$	Maximum strain	0.1411	1	105	Transverse matrix cracking	
15	$[0^\circ/90^\circ]$	Hoffman	0.1146	1	215	Fibre breakage	
	$[0^\circ/90^\circ/0^\circ]$	Puck	0.1402	3	13	Matrix crushing, mode C	
	$[0^\circ/90^\circ/0^\circ/90^\circ]$	Puck	0.1173	1	168	Matrix crushing, mode C	
	$[0^\circ/90^\circ/90^\circ/0^\circ]$	Maximum strain	0.1518	1	95	Transverse matrix cracking	
30	$[0^\circ/90^\circ]$	Hoffman	0.1429	1	69	Transverse matrix cracking	
	$[0^\circ/90^\circ/0^\circ]$	Puck	0.0926	3	13	Matrix crushing, mode C	
	$[0^\circ/90^\circ/0^\circ/90^\circ]$	Puck	0.1128	4	13	Matrix crushing, mode C	
	$[0^\circ/90^\circ/90^\circ/0^\circ]$	Puck	0.0894	4	13	Matrix crushing, mode C	
45	$[0^\circ/90^\circ]$	Hoffman	0.2238	1	13	Transverse matrix cracking	
	$[0^\circ/90^\circ/0^\circ]$	Puck	0.1758	3	213	Matrix crushing, mode C	
	$[0^\circ/90^\circ/0^\circ/90^\circ]$	Puck	0.0997	4	13	Matrix crushing, mode C	
	$[0^\circ/90^\circ/90^\circ/0^\circ]$	Puck	0.1493	4	13	Matrix crushing, mode C	

Table- 6. Uniformly distributed first ply failure load (FL) of angle ply skew plate, location of failure and first failed ply number (K)

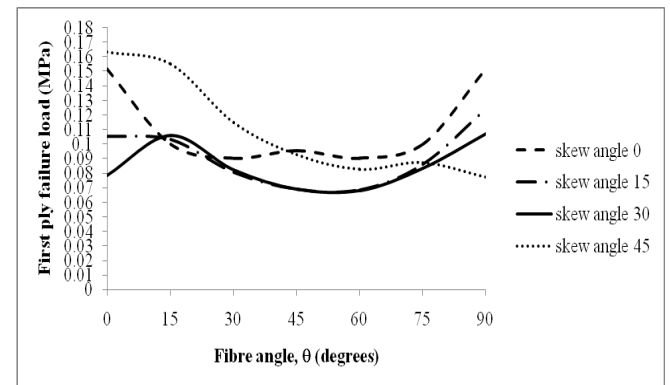
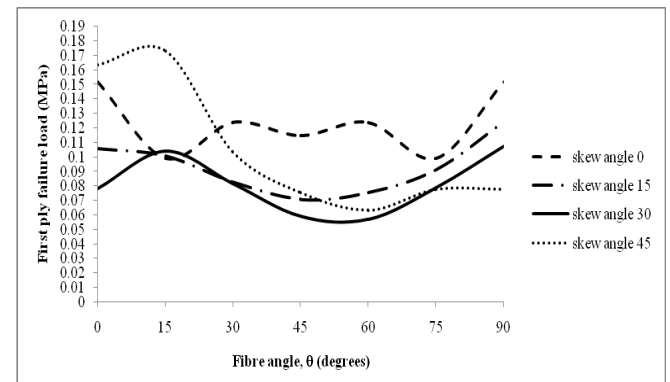
Skew angle, α (degree)	Stacking sequence	Governing failure criterion	FL (MPa)	K	Failure node	Failure mode/failure tendency
0	[45°/-45°]	Tsai-Wu	0.0515	1	121	Transverse matrix cracking
	[45°/-45°/45°]	Maximum strain	0.1018	3	197	Transverse matrix cracking
	[45°/-45°/45°/-45°]	Hoffman	0.0958	1	9	Transverse matrix cracking
	[45°/-45°/-45°/45°]	Maximum strain	0.1146	4	29	Transverse matrix cracking
15	[45°/-45°]	Maximum strain	0.0357	1	11	Transverse matrix cracking
	[45°/-45°/45°]	Maximum strain	0.0666	1	7	Transverse matrix cracking
	[45°/-45°/45°/-45°]	Maximum strain	0.0689	1	215	Transverse matrix cracking
	[45°/-45°/-45°/45°]	Maximum strain	0.0707	1	9	Transverse matrix cracking
30	[45°/-45°]	Maximum strain	0.0367	1	13	Transverse matrix cracking
	[45°/-45°/45°]	Maximum strain	0.0507	1	217	Transverse matrix cracking
	[45°/-45°/45°/-45°]	Maximum strain	0.0691	1	13	Transverse matrix cracking
	[45°/-45°/-45°/45°]	Maximum strain	0.0590	1	11	Transverse matrix cracking
45	[45°/-45°]	Maximum strain	0.0492	1	213	Transverse matrix cracking
	[45°/-45°/45°]	Maximum strain	0.0646	1	11	Transverse matrix cracking
	[45°/-45°/45°/-45°]	Maximum strain	0.0927	1	13	Transverse matrix cracking
	[45°/-45°/-45°/45°]	Maximum strain	0.0753	1	213	Transverse matrix cracking

clamped angle ply skew plate antisymmetric four layered laminate i.e. [45°/-45°/45°/-45°] shows better result in terms of first ply failure load values. Among clamped cross ply plates, highest failure load is exhibited by cross ply skew plate of skew angle 45° followed by cross ply rectangular plate and then skew plate of skew angle 15° and 30°. Whereas in case of clamped angle ply plates, highest failure load is shown by normal rectangular plate followed by 45° skew plate and then by skew plate of skew angle 15° and 30°. Skew angles considered in the present study are varied from 0° to 45° as for angles beyond 45° all other possibilities of skew angles are automatically encountered within this range if mirror image of the plate is considered. From the above tables it can be concluded that cross ply skew plate are better options than angle ply one in terms of first ply failure analysis. The highest failure load in case of clamped cross ply skew plate is obtained for [0°/90°] lamination for skew angle 45° which is about 2.41 times the highest failure load value among the angle ply skew plate options. Thus from engineering standpoint, for same consumption of material cross ply skew plate is preferable over angle ply ones.

In order to obtain maximum benefit from a given quantity of material, effect of variation in ply angles are studied and presented in Fig. 2 and Fig. 3. The ply angles $[\theta]$ are varied from 0° to 90° and the effect of ply orientation on first ply failure load is studied for $[\theta/\theta/\theta/\theta]$ and $[\theta/\theta/\theta/\theta]$ laminates. It can be seen from the Fig. 2 and 3, highest failure load is obtained when fibre angle (θ) is 0° or 90° for both four layered antisymmetric and symmetric laminates. This again infers that cross ply lamination is preferred over angle ply in case of skew plate configuration.

From Fig. 4 and 5, one can conclude that increasing the repetition of [0°/90°] units is not advantageous in increasing the first ply failure load value. On other hand for same material consumption first ply failure load value increases initially with the increase in [45°/-45°] units up to four layer laminates beyond which increasing the number of [45°/-45°]

unit has no such effect and first ply failure load value remains almost same.

**Fig. 2.** Variation of first ply failure load with fibre angle θ of $[\theta/-\theta/-\theta/-\theta]$ lamination for different skew plate options for CCCC boundary condition**Fig. 3.** Variation of first ply failure load with fibre angle θ of $[\theta/-\theta/-\theta/\theta]$ lamination for different skew plate options for CCCC boundary condition

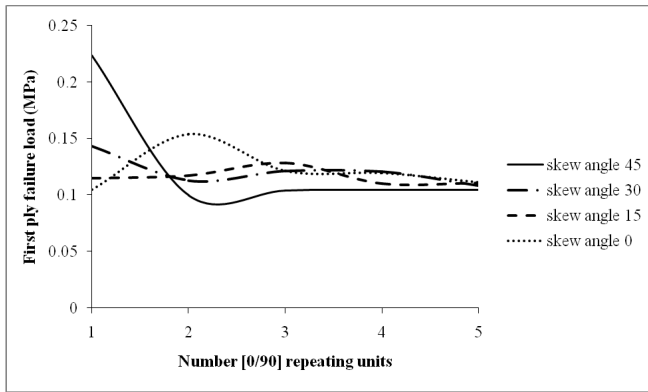


Fig. 4. Variation of first ply failure load with [0/90] repeating units for different skew plate options

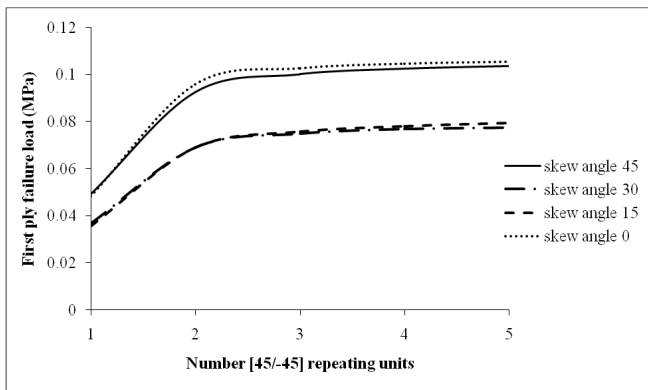


Fig. 5. Variation of first ply failure load with [45/-45] repeating units for different skew plate options

5. Conclusions

Following conclusions are made from the above study

- The present finite element code can efficiently predict the first ply failure loads of laminated composite skew plate as it is evident from the solutions of the benchmark problem.
- Cross ply skew plates fail by matrix cracking or matrix crushing mode of failures. On other hand angle ply skew plates always failed by transverse matrix cracking.
- The clamped cross ply skew plates are convincingly better than angle ply skew plates in terms of first ply failure loads.

- As the boundary condition is fully clamped so failure happens at supports or elements near supports in almost all cases.
- For both angle ply and cross ply skew plates 45° skew angle gives the highest failure load value. For cross ply antisymmetric two-layered laminate [0°/90°] shows the best performance and for angle ply skew plate antisymmetric four layered [45°/-45°/45°/-45°] shows the best performance in term of first ply failure.
- The repetition of [45°/-45°] laminate up to four is advantageous for fabricating new angle ply laminates.

Disclosures

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