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Application of novel refined mixed finite element method in the analysis of composite laminated beams

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Abstract

This paper shows the application of refined finite element model developed by authors, which incorporate all the elastic equation that are, equilibrium equation, constitutive equation and kinematic equation to be satisfied explicitly at nodes. The present method considers the equilibrium equation for the refinement of the approximation function. Authors used the concept of mixed finite element model by choosing stress and displacement as the primary variable, along with the equilibrium equation as a refinement on approximation function. Ramtekkar [1] in his model restrict the arbitrary choosing of stress function by selecting the stress variation from displacement derived space, by imposing the elastic relations on the variation and then used the Minimum Potential Energy Principle to obtain the governing differential equation. In present work, authors satisfy the equilibrium equation at nodes also by considering the body forces as variables inthe formulation which enables the model to satisfy the equilibrium equation explicitly at the node. The formulation is validated by the elastic beam problem solvedby Pagano [2] for all three cases. Results obtained are very accurate and also shows faster convergence over the Ramtekkar [1] model.
Keywords: Mixed finite element method, composite laminates,

1. Introduction

Mixed finite element formulation started gaining popularity after the pioneering work in this field by Ressiner [3]. In mixed finite element formulation, stress quantities are also considered as a primary variable along with the displacement quantities and solution is achieved through the stationarity principle of Hellinger-Reissner's. This formulation proofs beneficial in cases wherethe determination of stress is important. In displacement-based formulation, stress quantities are derived by operating kinematic relation and constitutive relation. This methodleads to significant approximation in the calculation of stresses. Displacement based finite element formulations are not able to control the specific stress values in the boundary of the domain.Determination of interlaminar variation is very important in case of composite laminates as interlaminar spaces are weak zone.

To find the through thickness variation of stress in composite laminates many approaches have been usedby various researchers, many of them considered the composite laminate as equivalent single layer and analysed the laminates which gave better results for the global behaviour of the laminates , some of the ESL theories are First order shear deformation theory higher-order shear deformation theories by Reddy[4] and Kant[5]–[7]. To analyse the composite laminates for through thickness variation of stresses Layer wise theories has been developed in which mixed theories , zig-zag theory by Carrera [8]–[11]partial discretisation approach by Kant[5], which could be grouped under the axiomatic theory.

A generalised 2-dimensional plane stresses finite element formulation was developed by Ramtekkar & Desai [1]. They developed a 6 node mixed finite element model to analyse the laminated composite beam. This element contains u, w, σ_{zz}, τ_{xz}as primaryvariables and used minimum potential energy principle to formulate the governing differential equation. Benefit of this formulation is that it could directly calculate stress values and also could specify the predefined stress values in theformulation to maintain the traction free surface. This formulation also usesminimization principle which is better than the stationarity (Hellinger-Reissner) principle.

In present work which is a refinement over Ramtekkar [1] model, authors consider the equilibrium as a constraint on the approximation function. This formulation consider u, w, σ_{zz}, τ_{xz}as primary variables along with the body forcesp_x, p_z as refinement, term to improve the results. This gives the 6 quantities per node which could reduce to 4, after the refinement of body forces part. This refinement could be incorporated in the formulation by the static condensation.

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The present formulation includes equilibrium consideration along with the constitutive and kinematic relation. Henceall the elasticity condition has been satisfied with this formulation, which develops results very close to an elastic solution at nodes. Other than this, present formulation gives fast convergence than Ramtekkar [1] formulation.

2. Formulation

To analyze the composite laminates beam in 2-dimesion 6 node element has been formulated as shown in Fig 2. u, w, σ_{zz}, τ_{xz}were chosen as primary variables at each node, which ultimately leads to 24 variables in each element. Equilibrium equation was taken for the refinement of approximation function, which could be achieved through static condensation process. For the formulation of the finite element stiffness matrix approximation function were chosen as shown in Eq.-1

$$u_k(x,z) = \sum_{i=1}^{3} g_i a_{0ik} + z\sum_{i=1}^{3} g_i a_{1ik} + z^2\sum_{i=1}^{3} g_i a_{2ik} + z^3\sum_{i=1}^{3} g_i a_{3ik} + z^4\sum_{i=1}^{3} g_i a_{4ik} + z^5\sum_{i=1}^{3} g_i a_{5ik} \tag{1}$$

where:

$$g_1 = \frac{\xi}{2}(\xi-1), g_2 = (1-\xi^2), g_3 = \frac{\xi}{2}(1+\xi) \tag{2}$$

$$u_1(x,z) = u(x,z), u_2(x,z) = w(x,z) \tag{3}$$

Variation of stress were derived from the Eq -1, by using the kinematic and constitutive relation, and equilibrium equation was imposed in the formulation by considering the body forces and applying refinement through static condensation.

Which leads to final equation to be solved of the form as shown in Eq-4.

$$[K]\{D\} = \{F\} \tag{4}$$

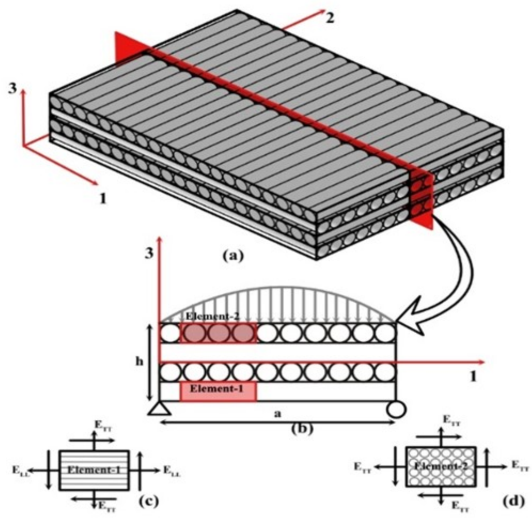

Fig 1Schematic diagram of composite laminate simply supported beam subjected to transverse sinusoidal loading (a) Laminated cross-ply beam with reference axes, (b) representative section of the 3D beam in 2 dimension with support condition and loading direction, (c) representative element with fibre orientation, (d) representative element with fibre orientation to address cross-ply

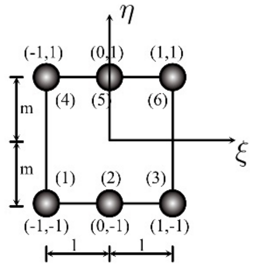

Fig 2 2-Dimensional 6 node plane stress element

where:

$$[K] = \sum [K]_e \, ; [K]_e = \int_{-L_x}^{L_x}\int_{-L_y}^{L_y}\int_{-L_z}^{L_z} [B]^T [Q][B] dxdydz \tag{5}$$

$$\{D\}^T = \sum \{D\}_e^T \;\; ; \{D\}_e^T = \{u_1, w_1, \sigma_{zz_1}, \tau_{xz_1} \ldots u_6, w_6, \sigma_{zz_6}, \tau_{xz_6}\}^T \tag{6}$$

$$[F] = \sum [F]_e \, ; [F]_e = \int_{-L_x}^{L_x}\int_{-L_y}^{L_y}\int_{-L_z}^{L_z} [N]^T \{p_b\} dxdydz + \iint_{\Sigma} [N]^T\{p_t\} ds \tag{7}$$

$[B]^T$ = Strain matrix
$[Q]$ = Constitutive matrix
$[N]^T$ = Interpolation matrix
$\{p_b\}$,$\{p_t\}$ = Body force vector and traction force vecto

3. Numerical results and discussion

To validate the formulation, elastic solution of a 2D composite laminate beam by Pagano [2] and solution given by Ramtekkar [1]has been compared with the present formulation. The first example is of a unidirectional laminated beam subjected to sinusoidal loading in the transverse direction. Material properties of the laminates are given in Table 1 which has been taken from the Pagano [2]. The boundary condition for the different primary variables at the different location has been given in Table 2. In the

Table 1: Material properties for the problem (Pagano [2])

E_L=25x10^6 psi	E_T=1x10^6 psi	G_{LT}=5x10^5 psi
G_{TT}=2x10^5 psi	$\nu_{LT} = \nu_{TT} = 0.25$	

Table 2: Boundary conditions for a different problem

Problem	Location	Primary variable			
		u	**w**	τ_{xz}	σ_z
1	X=0	-	0	-	-
	X=a/2	0	-	0	-
	Z=+d/2	-	-	0	$\tilde{q}(x)$
	Z=-d/2	-	-	0	0
2	X=0	-	0	-	-
	X=a/2	0	-	0	-
	Z=+d/2	-	-	0	$\tilde{q}(x)$
	Z=-d/2	-	-	0	0

series of the problem first, we solve the single layer laminated simply supported composite beam, subjected to sinusoidal load in the positive z direction and normalized parameter were plotted and compare with the benchmark solution in Fig 3. In the second case two layered cross ply (0°/90°) laminated unsymmetrical composite beam as shown in Fig 1, has been solved to get the variation of normalised transverse displacement $\overline{W}$, in- plane normal stress ($\overline{\sigma}_x$), and transverseshear stress ($\overline{\tau}_{xz}$) as shown in Table 3, of double layered laminates under cylindrical bending and results obtained were plotted in Fig 4. Six node FE model with the u, w, σ_{zz}, τ_{xz} as primary variable in each nodeand body forces p_x, p_z as refinement parameter is used to solvethe above problem, and results obtained arecompared with the other benchmark solution available.

After, comparing the result from the benchmark solution we could observe that present formulation gives better results with the fewer element, hence reducing the matrix size upto two third of Ramtekkar [1], formulation which is markable saving in computational cost.

Table 3: Non-dimension alization coefficients

$$s = \frac{a}{d}; \quad \overline{Z} = \frac{Z}{d}; \quad \overline{U} = \frac{E_2 U}{d q_0}; \quad \overline{W} = \frac{100 E_2 W}{q_0 d s^4}$$

$$\overline{\sigma}_x = \frac{\sigma_x}{q_0} \; ; \; \overline{\sigma}_z = \frac{\sigma_z}{q_0} \; ; \; \overline{\tau}_{xz} = \frac{\tau_{xz}}{q_0};$$

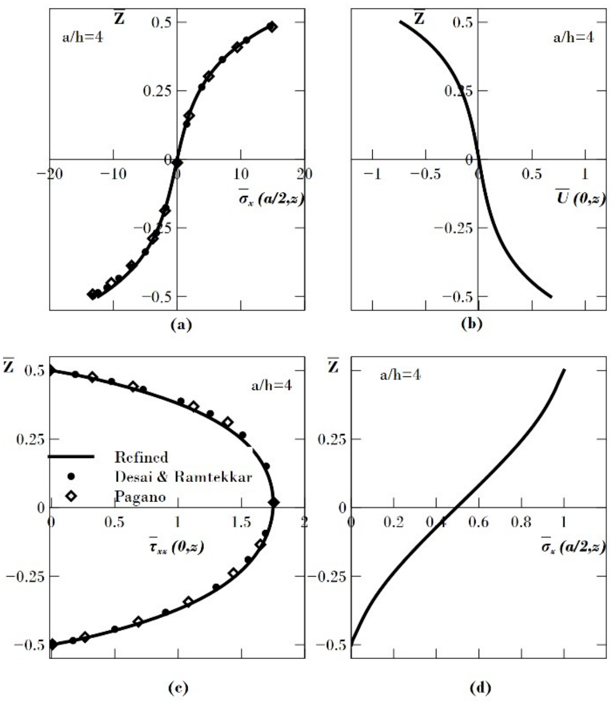

Fig 3Variation of normalized (a) in-plane normal stress ($\overline{\sigma}_x$); (b) in-plane displacement ($\overline{U}$); (c) transverse shear stress ($\overline{\tau}_{xz}$); (d) transverse normal stress ($\overline{\sigma}_z$) through the thickness of a simply supported laminated beam with s=4, under sinusoidal loading

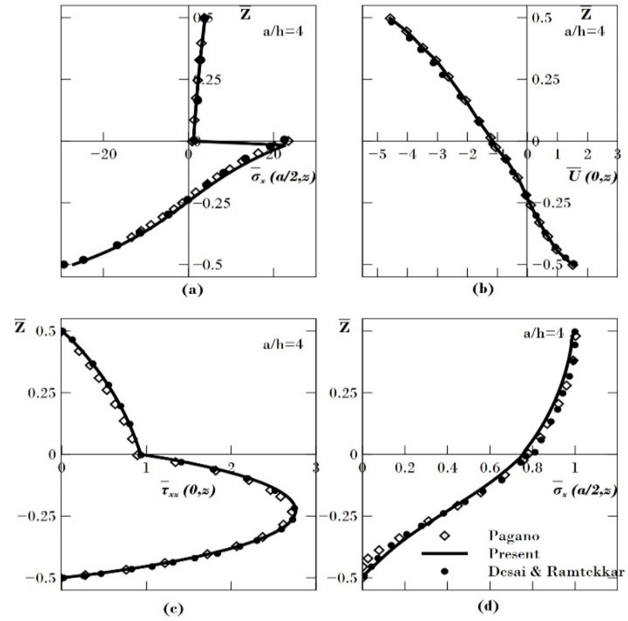

Fig 4 Variation of normalized (a) in-plane normal stress ($\overline{\sigma}_x$); (b) in-plane displacement ($\overline{U}$); (c) transverse shear stress ($\overline{\tau}_{xz}$); (d) transverse normal stress ($\overline{\sigma}_z$) through the thickness of a simply supported laminated beam with s=4 and laminated scheme (0°/90°), under sinusoidal loading

Table 4: Comparison of the maximum transverse displacement, the in-plane normal and transverse shear stresses for simply supported laminated beam under sinusoidal loading (laminate scheme: 0°/90°)

Prob	Stress/ Displacement	Present Analysis	Pagano [2]	Ramtekkar [1]
2	$\overline{\sigma}_x(\frac{a}{2},\frac{d}{2})$	3.8352	3.8359	3.8247
	$\overline{\sigma}_x(\frac{a}{2},-\frac{d}{2})$	-29.7892	-29.9745	-29.9383
	$\overline{\tau}_{xz}(max.)$	2.7375	2.7300	2.7500
	$\overline{W} \;\; (\frac{a}{2},0)$	4.7952	4.7675	4.7636

4. Conclusion

A refined FE model has been presented. This formulation shows excellent agreement with the elastic solution and numerically comparable with Ramtekkar [1] model. This model is unique in the sense that it explicitly incorporates refinement from equilibrium equation, constitutive equation and kinematic equation. This formulation is giving accurate results in the lesser number of elements than Ramtekkar [1] model. Present formulation has been develop dusing the principle of minimum potential energy which is minimization principle and taking stress parameters along with the displacement as primary variable which allows the freedom to specify the specific stress values. This model gives the benefit of mixed formulation with the advantage of minimization principlewhich could give the stable solution. This refinement do not increase the number of primary variables but reduce the number of element required to obtain the result more close to benchmark results.

Disclosures

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References

[1] Y. M. Desai and G. S. Ramtekkar, 'Mixed finite element model for laminated composite beams', *Struct. Eng. Mech.*, vol. 13, no. 3, pp. 261–276, 2002, doi: 10.12989/sem.2002.13.3.261.

[2] N. J. Pagano, 'Exact Solutions for Composite Laminates in Cylindrical Bending', *J. Compos. Mater.*, vol. 3, no. 3, pp. 398–411, Mar. 1969, doi: 10.1177/002199836900300304.

[3] E. Reissner, 'On a Variational Theorem in Elasticity', *J. Math. Phys.*, vol. 29, no. 1–4, pp. 90–95, Apr. 1950, doi: 10.1002/sapm195029190.

[4] J. N. Reddy, 'A generalization of two-dimensional theories of laminated composite plates', *Commun. Appl. Numer. Methods*, vol. 3, no. 3, pp. 173–180, May 1987, doi: 10.1002/cnm.1630030303.

[5] T. Kant, S. S. Pendhari, and Y. M. Desai, 'A General Partial Discretization Methodology for Interlaminar Stress Computation in Composite Laminates', p. 28, 2007.

[6] T. Kant, S. S. Pendhari, and Y. M. Desai, 'A Novel Finite-Element–Numerical-Integration Model for Composite Laminates Supported on Opposite Edges', *J. Appl. Mech.*, vol. 74, no. 6, pp. 1114–1124, Nov. 2007, doi: 10.1115/1.2722770.

[7] T. Kant, S. S. Pendhari, and Y. M. Desai, 'An Efficient Semi-Analytical Model for Composite and Sandwich Plates Subjected to Thermal Load', *J. Therm. Stress.*, vol. 31, no. 1, pp. 77–103, Dec. 2007, doi: 10.1080/01495730701738264.

[8] E. Carrera, 'Transverse Normal Stress Effects in Multilayered Plates', *J. Appl. Mech.*, vol. 66, no. 4, pp. 1004–1012, Dec. 1999, doi: 10.1115/1.2791769.

[9] E. Carrera, 'Theories and finite elements for multilayered, anisotropic, composite plates and shells', *Arch. Comput. Methods Eng.*, vol. 9, no. 2, pp. 87–140, Jun. 2002, doi: 10.1007/BF02736649.

[10] E. Carrera, 'Developments, ideas, and evaluations based upon Reissner's Mixed Variational Theorem in the modeling of multilayered plates and shells', *Appl. Mech. Rev.*, vol. 54, no. 4, pp. 301–329, Jul. 2001, doi: 10.1115/1.1385512.

[11] E. Carrera, 'An assessment of mixed and classical theories on global and local response of multilayered orthotropic plates', *Compos. Struct.*, vol. 50, no. 2, pp. 183–198, Oct. 2000, doi: 10.1016/S0263-8223(00)00099-4.