

Effect of snap-through force location on morphing of bistable variable stiffness laminates

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Paper ID - 010103

Abstract

Unsymmetric composite laminates exhibit two or more stable states due to the presence of residual thermal stresses induced during the curing process. The paper aims to exploit the possibility of assessing the morphing behaviour of bistable composite laminates using semi-analytical and numerical models. A Rayleigh-Ritz based semi-analytical model is used to investigate the variable stiffness (VS) bistable laminates generated using curvilinear fibre alignments. The obtained results are verified using a fully non-linear numerical model developed in a commercially available finite element package. In order to identify the influence of loading location on the snap-through behaviour, a parametric study is proposed in the present research. Snap-through is achieved through transverse point loads. The novelty of this work resides on the investigation of load application location on the snap-through action where the transverse load is applied at various locations on the lamina surface. Earlier research on the morphing of bistable square composites is mainly on the application of concentrated load at the centre of the laminate geometry, which is considered as an ideal location for snap-through action. The designer may also have to choose other locations for the application of snap-through loads since the lamina centre may not be appropriate always in structural applications. Four different loading locations are selected for the present analysis, and snap-through loads are calculated to check the effect of loading location. Parametric studies are performed using the proposed semi-analytical and numerical models to investigate the effect of selected loading point positions on the snap-through process. The analysis is extended to a selected VS laminate family to understand the influence of loading point positions on the curvilinear fibre alignments.

Keywords: Bistability, morphing, snap-through, numerical analysis, laminate

1. Introduction

Morphing multistable structures are treated as the adoptable smart structures, which can show reconfigurable stable shapes with respect to the changes in the surrounding loads. Shape morphing structures exhibit various equilibrium stable shapes and require actuation energy for the shape transition during its structural performance. Multistability in the mechanics context can be explained using the principle of the minimum total potential energy of the system. The equilibrium states of a multistable system can be determined by the minima's in the potential energy. If the potential energy of the system has only one minimum in the domain, then it is referred to as a monostable system, whereas bistable system has two local minima of potential energy separated by a peak of local maxima. On the other hand, when the potential energy function has two or more than two minima in its domain, then the system is defined as multistable. A multistable system can have a transition from one equilibrium state to the other if it is supplied enough energy to overcome the barrier between the states. The

transition between shapes occurs through a dynamic jump phenomenon known as a snap-through event and it is highly nonlinear in nature. Multistable structures have attracted a lot of attention in the recent past, particularly in applications like morphing and deployable structures [1-4], and energy harvesters [5-6]. The attraction derives from their capability to achieve multiple configurations with large displacements using relatively low energy input while being lightweight, mechanically simple, and stiff enough to be part of the load bearing structure.

The basic requirement in the design of a multistable laminate is to incorporate residual stress in the design such that multiple strain energy minimas are possible. Residual stresses can be imparted to structural components in several categories; two of them are namely thermally cured methods and mechanically induced methods [7]. One of the classical examples for thermally cured multistability can be identified in curing shapes of unsymmetrical laminates [8]. Hyer

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continued his earlier attempts to develop a theory to explain the behaviour of the cured shapes of thin unsymmetric laminates using von-Karman geometric nonlinearities with classical lamination theory in order to identify the room-temperature shapes [9]. Temperature curvature relations investigations by Hamamoto and Hyer [10] identified the influence of imperfection on the behaviour of cross-ply bistable laminates by including imperfections like the orientation of lamina, thickness of lamina and the uniformity of heat distribution during cooling. It is concluded that imperfections are needed to avoid the occurrence of saddle unstable shape upon curing process. Hyer's work is later extended by Dang and Tang for angle-ply laminates [11] and noticed the presence of a twist curvature in cured shapes in addition to the curvature in the principal directions. Polynomial approximations used by Dang and Wang [11] are subsequently modified by various researchers [12]-[14]. Numerous works on semi-analytical models have been reported in the literature to analyse the cured room-temperature shapes of unsymmetric laminates [15-18]. Analytical models developed by various researchers explained the computational cost difficulties in predicting the exact cured shapes of the unsymmetric laminates, especially at its boundaries. To predict the shapes and snap through loads accurately, higher order of polynomial functions are required in the analytical model which increases the computational cost. Besides the analytical approaches, researchers have also employed the finite element (FE) method to study the behaviour of bistable plates. With refined FE models [19-20], it is possible to predict the bistable shapes with higher accuracy, and they are closer to the experimentally observed shapes than the predictions from analytical models developed so far. At the same time, some imperfections like force, moment or small changes in the geometry must be applied to coax the FE model to converge to one of the stable solutions.

Snap-through between the bistable shapes can be triggered using different techniques. Dano and Hyer [21] successfully investigated static snap-through mechanism by applying corner point forces on the cured shapes of unsymmetric cross-ply laminates. Later, Schlecht and Schulte [20] extended the studies with edge forces to trigger the snap-through from one position to the other with a fixed boundary condition at the centre point. The main challenge faced by all these researches is the authority that is enough to trigger the snap-through motion is found to be high. As a solution, recent studies by Halder et al. [16] reported the potential of tailoring snap-through loads by choosing curvilinear fibre alignments without compromising the out-of-plane displacements. VS composite similar to those of unsymmetric straight fibre laminates exhibits multistable shapes when cured from high temperature to room temperature. Halder et al. [16] showed that the bistable cylindrical shapes similar to the ones conventionally obtained from cross-ply laminates can also be attained using VS laminates, but with the possibility to reduce the snap-through loads, which is advantageous in morphing applications.

The present research aims to develop a semi-analytical model to predict the bistable behaviour of cured shapes of

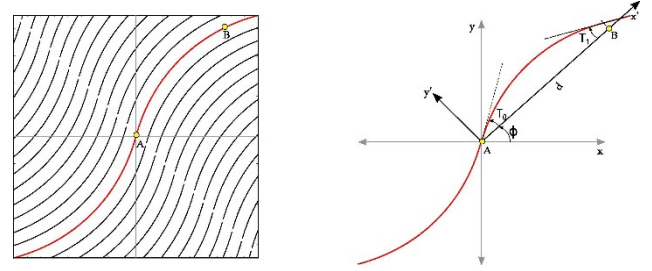


Figure 1.Parameters defining the VS path[16]

Table-1. Laminate families considered in the study

VS Family	Description
VS-1	45<15/75> / 45<-15/-75>
VS-2	45<30/60> / 45<-30/-60>
VS-3 or [0/90]	45<45/45> / 45<-45/-45>
VS-4	45<60/30> / 45<-60/-30>
VS-5	45<75/15> / 45<-75/-15>

VS unsymmetrical laminates. A corresponding FE model is also developed to compare the results from the semi-analytical model. In order to identify the influence of location of applied external loads on the snap-through behaviour, a parametric study is proposed in this work. Four selected points are taken for the study, and the details are depicted in the subsequent sections. The studies are further extended to a selected VS family to explore possibility of tailoring snap-through loads by altering the VS parameters.

2. Variable stiffness laminate model

VS laminates investigated in this work are modelled based on a curvilinear fiber path description proposed by Gürdal et al. [22].The fiber orientation defined is given as follows.

$$\theta(x') = \theta + \frac{(T_1 - T_0)|x|}{d} + T_0 \quad (1)$$

$$\text{where, } x' = x \cos \phi + y \sin \phi \quad (2)$$

The standard notation to define a particular VS laminate with the above-mentioned three parameters is given as follows; $\phi < T_0 | T_1 >$. T_0 is defined as the angle at the center of the plate and T_1 is defined as the angle at the edge of the plate. The present study has been proposed with $\phi = 45^\circ$, thereby generating the new axes (x' and y') which makes an angle of 45° with the Cartesian coordinate axes. Laminates selected for the present study are given in Table 1.

3. Semi-Analytical Modelling

Investigations on the semi-analytical (SA) formulation of bistable laminates are described in this section. The mathematical formulations are based on von Kármán extension of classical lamination theory (CLPT).

4.1 Curing stage of laminate

The displacement vector components are defined as follows:

$$\begin{aligned}
u(x, y, z) &= u_0(x, y) - z \frac{\partial w_0}{\partial x} \\
v(x, y, z) &= v_0(x, y) - z \frac{\partial w_0}{\partial y} \\
w(x, y, z) &= w_0(x, y)
\end{aligned}$$

(1) u_0, v_0 and w_0 are displacements at the mid-plane. The strain tensors are represented as follows:

$$\begin{aligned}
\epsilon_{xx} &= \frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \\
\epsilon_{yy} &= \frac{\partial v}{\partial y} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 \\
\gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y}
\end{aligned} \quad (2)$$

The three curvature fields are represented as follows.

$$\mathbf{k} = \begin{bmatrix} k_{xx} \\ k_{yy} \\ k_{xy} \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 w_0}{\partial x^2} \\ \frac{\partial^2 w_0}{\partial y^2} \\ \frac{\partial^2 w_0}{\partial x \partial y} \end{bmatrix} \quad (3)$$

Force and moments resultant equations are:

$$\begin{bmatrix} \mathbf{N} \\ \mathbf{M} \end{bmatrix} = \begin{bmatrix} \mathbf{A}(x, y) & \mathbf{B}(x, y) \\ \mathbf{C}(x, y) & \mathbf{D}(x, y) \end{bmatrix} \begin{bmatrix} \boldsymbol{\epsilon} \\ \mathbf{k} \end{bmatrix} - \begin{bmatrix} \mathbf{N}^{th} \\ \mathbf{M}^{th} \end{bmatrix} \quad (4)$$

$\mathbf{N}$ and $\mathbf{M}$ are total force and moment resultants respectively. In the formulation, $\mathbf{ABD}$ matrix is constant for conventional cross-ply laminates; whereas in VS shells, $\mathbf{ABD}$ matrix is a function of coordinates considered.

The total potential energy of the bistable shell during cool-down stage is given by:

$$\Pi_L = \iint \left(\frac{1}{2} \begin{bmatrix} \boldsymbol{\epsilon} \\ \mathbf{k} \end{bmatrix}^T \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \boldsymbol{\epsilon} \\ \mathbf{k} \end{bmatrix} - \begin{bmatrix} \mathbf{N}^{th} \\ \mathbf{M}^{th} \end{bmatrix}^T \begin{bmatrix} \boldsymbol{\epsilon} \\ \mathbf{k} \end{bmatrix} \right) dx dy \quad (5)$$

Where the subscript L represent laminate considered in the study. $\mathbf{N}^{th}$ and $\mathbf{M}^{th}$ shows the temperature curing effect and are given as:

$$\{\mathbf{N}^{th}(x, y)\} = \sum_{k=1}^n \Delta T \{\tilde{\mathbf{Q}}\}_k \{\alpha\}_k (z_k - z_{k-1}) \quad (6)$$

$$\{\mathbf{M}^{th}(x, y)\} = \frac{1}{2} \sum_{k=1}^n \Delta T \{\tilde{\mathbf{Q}}\}_k \{\alpha\}_k (z_k^2 - z_{k-1}^2) \quad (7)$$

4.2 Rayleigh- Ritz Approximations

From Eq. (6), strain component can be identified as:

$$\begin{aligned}
\epsilon_{xx}^0 &= A^{-1} N + A^{-1} \mathbf{N}^{th} - A^{-1} B \mathbf{k} \\
\epsilon_{xx}^0 &= \epsilon_{xx}^m + A^{-1} \mathbf{N}^{th} - A^{-1} B \mathbf{k}
\end{aligned} \quad (8)$$

Where $A^{-1} N$ is considered as a membrane strain field. With the kinematic and constitutive relations, admissible shape functions for the mid-plane strains and displacements are required to capture the equilibrium shapes of unsymmetric bistable laminates. In this paper, instead of taking the mid-plane strains as the primary variables, stretching strains are assumed.

$$\begin{aligned}
\epsilon_{xx}^m &= c_1 + c_2 y^2 + c_3 y^4 + c_4 y^6 + c_5 x^2 y^2 \\
\epsilon_{yy}^m &= c_6 + c_7 x^2 + c_8 x^4 + c_9 x^6 + c_{10} x^2 y^2
\end{aligned} \quad (9)$$

From the assumed function, using Eq. (8), the extensional mid-plane strains are calculated.

A quadratically varying curvature polynomial is chosen for the out-of-plane displacement function that satisfies the fixed centre geometric boundary conditions of the composite laminate.

$$w_0(x, y) = c_{11} x^2 + c_{12} y^2 + c_{13} x^4 + c_{14} y^4 + c_{15} x^2 y^2 \quad (10)$$

Using the expression for extensional strain one can compute the in-plane displacements:

$$u_0(x, y) = \int \left(\epsilon_{xx}^0 - \frac{1}{2} \left(\frac{\partial w_0}{\partial x} \right)^2 \right) dx$$

$$v_0(x, y) = \int \left(\epsilon_{yy}^0 - \frac{1}{2} \left(\frac{\partial w_0}{\partial y} \right)^2 \right) dy \quad (11)$$

The shear strain can be simply calculated from the equation below

$$\gamma_{xy}^0 = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x} \frac{\partial w_0}{\partial y} \quad (12)$$

4.3 Work done by external loads

An actuation force is required to trigger snap-through from one state to another. In the present study, concentrated forces are imposed on shell corners. The virtual work of applied forces is calculated as:

$$\begin{aligned}
W_F &= 4f \left\{ c_{11} \left(\frac{L_x}{2} \right)^2 + c_{12} \left(\frac{L_y}{2} \right)^2 + c_{13} \left(\frac{L_x}{2} \right)^4 + c_{14} \left(\frac{L_y}{2} \right)^4 \right. \\
&\quad \left. + c_{15} \left(\frac{L_x}{2} \right)^2 \left(\frac{L_y}{2} \right)^2 \right\} \quad (13)
\end{aligned}$$

The calculated external energy is included for subsequent calculations.

4.4 Stability of laminates

Rayleigh-Ritz approach is used on cool-down and snap-through stages ($\delta \Pi_L = 0$). Total potential energy is a function of unknowns from displacement functions ($\mathbf{c}_i$, $i = 1$ to n as defined in Eq. (10)). For a given ΔT , the potential energy evaluated in terms of $\mathbf{c}_i$'s as.

$$\Pi \approx \Pi_N(\mathbf{c}), \mathbf{c} = \{c_i\}, i = 1, 2, \dots, n \quad (14)$$

Deformed configurations of the bistable laminates at the end of each step are evaluated by the minimising the total energy for the system. The non-linear equation system obtained (Eq. 10) is examined with the Newton-Raphson technique.

$$\frac{\partial \Pi_N(\mathbf{c})}{\partial c_i} = 0 \quad (15)$$

Jacobian matrix ($\mathbf{J}$) can be used for the evaluation of stability.

$$\mathbf{J} = \frac{\partial^2 \Pi_N}{\partial c_i \partial c_j}, i, j = 1, 2, 3, \dots, n \quad (16)$$

Hence, an equilibrium configuration is stable if and only if the corresponding Jacobian matrix (16) is positive definite, and is unstable otherwise. All the algebraic computations are accomplished using the software, Mathematica

4. Numerical Analysis

A geometrically non-linear finite element model is developed using commercial software (AbaqusTM) to verify the semi-analytical model. Imperfections are introduced to the laminate model to coax the FE model to converge to one of the stable solutions. Composite shell elements of type S4R (Four-node quadrilateral shell element) are used to carry out FE analysis. Mesh convergence study is carried out and based on which mesh characteristics with suitable mesh size is chosen. The details of the steps in the analysis are described as follows.

Initial Step in the modelling: A curing temperature of 140°C is imposed on the laminates during this stage. Central node of the laminate is considered as fixed in the analysis. Temperature field command in AbaqusTM, which simulates the curing effects

Cool-down step: The temperature field of laminate part is cooled down to room temperature by imposing a temperature difference of -120°C. This temperature difference makes the composite laminates to deviate from its original shape to warp into one of the stable shapes.

Snap-through and stability check of snap-through: Static snap-through process is performed by applying transverse loads to the surface of the cured shapes of the laminate. Stability check is imposed at the end of snap-through to make sure that the shape remains unchanged even after removing the external loads.

5. Example considered for the validation

A square laminate of side length 135mm is taken for the present study. The geometry considered for the analysis is given in Figure 2. The properties of the lamina are given in Table 2.

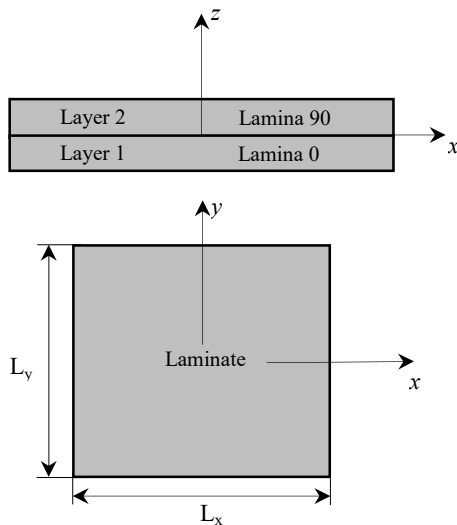


Figure 2. Laminate geometry considered for study

Table-2. Material properties of lamina.

Property	Value
E_{11}	140 GPa
E_{22}	13 GPa
G_{12}	6.6 GPa
ν	0.3
α_{11}	$-0.8 \times 10^{-6}/^{\circ}\text{C}$
α_{22}	$29 \times 10^{-6}/^{\circ}\text{C}$
t	0.149 mm

6. Results and discussion

In order to validate the developed models (section 3 and 4), cool-down and snap-through stages are performed on the selected lamina geometry (section 5). The comparison of out-of-plane displacements for selected VS families for the first stable shape is given in Table 3. Figure 3 shows the out of plane displacement comparison of one of the bistable shape generated from [0/90] laminate (at a section, $y=0$). Although there is a small discrepancy of deformed shapes at the edges, the overall deformed shape predicted by the semi-analytical model is in good agreement with the results from numerical model. Out of plane displacement comparison of VS bistable shapes using semi-analytical model and numerical model are given in Figure 4 and Figure 5 respectively. There is a noticeable change in the bistable

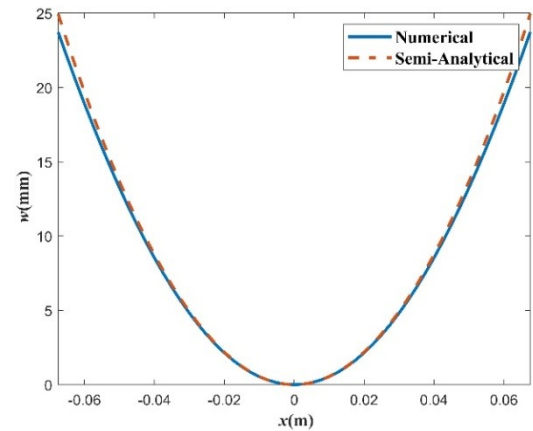


Figure 3. Out-of-plane displacement comparison of [0/90] bistable shapes at $y=0$

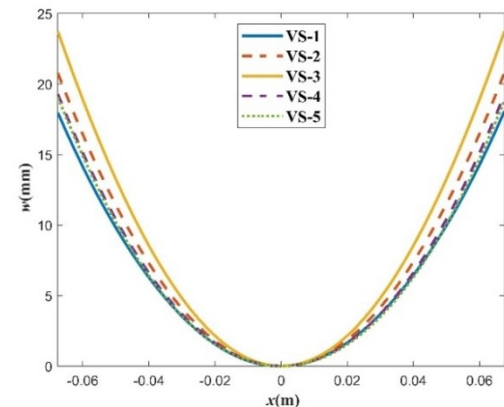


Figure 4. Out of plane displacement comparison of VS bistable shapes at $y=0$, using numerical model

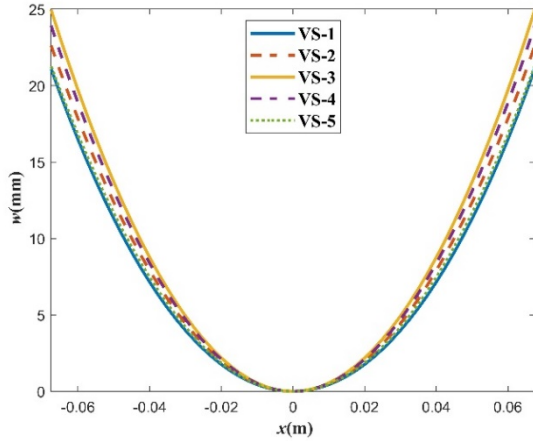


Figure 5. Out of plane displacement comparison of VSbistable shapes at $y=0$, using semi-analytical model

Table-3. Comparison of out-of-plane displacements between semi-analytical and numerical models

VS family	Out-of-plane displacements at corner (mm)	
	Semi-Analytical	Numerical
VS-1	20.06	18.98
VS-2	22.87	19.83
VS-3	24.71	21.78
VS-4	24.07	21.35
VS-5	21.54	20.87

shapes obtained from different selected VS laminates. This may be due to local variations in the generated residual stresses in the VS laminates due to variation in the material properties at each point (For example, Figure 6 and Figure 7 shows the A_{11} of [0/90] Laminate and VS-4 respectively, and Figure 8 and Figure 9 shows the M_{xx} of [0/90] Laminate and VS-4 respectively).

The comparison of snap-through loads between the semi-analytical and numerical model are given in Table 4. The results predicted by the semi-analytical model is in good agreement with the numerical results. The percentage difference between numerical and semi-analytical results are within 1 - 8% for VS laminates. It is observed that replacing conventional straight fibres with curvilinear variable stiffness laminates results in lower snap-through loads without much differences on the out-of-plane displacements. VS laminates like VS-1 and VS-2 and VS-5 shows the lower snap-through loads than the straight fibre ([0/90]) laminates.

Table-4. Comparison of snap-through load between semi-analytical and numerical models

VS family	Snap-through force (N)	
	Semi-Analytical	Numerical
VS-1	1.68	1.72
VS-2	2.48	2.67
VS-3	3.00	3.17
VS-4	3.16	3.28
VS-5	2.36	2.43

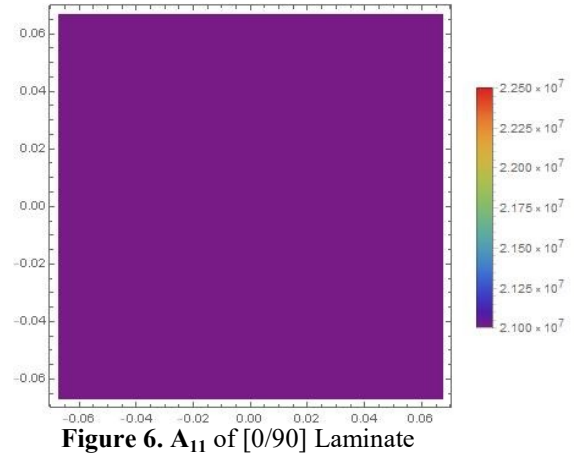


Figure 6. A_{11} of [0/90] Laminate

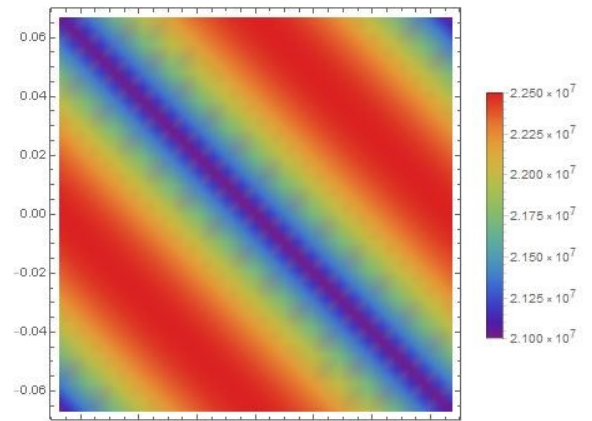


Figure 7. A_{11} of VS-4 Laminate

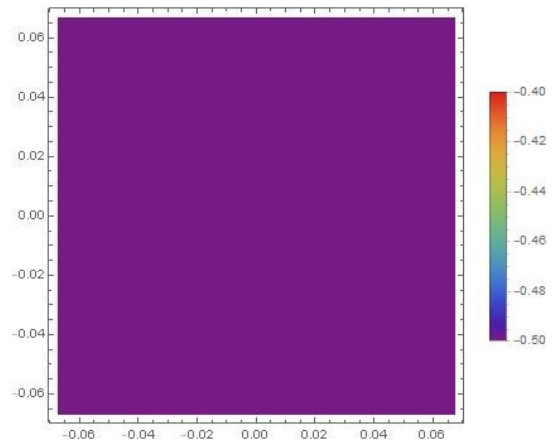


Figure 8. M_{xx} of [0/90] Laminate due to Curing

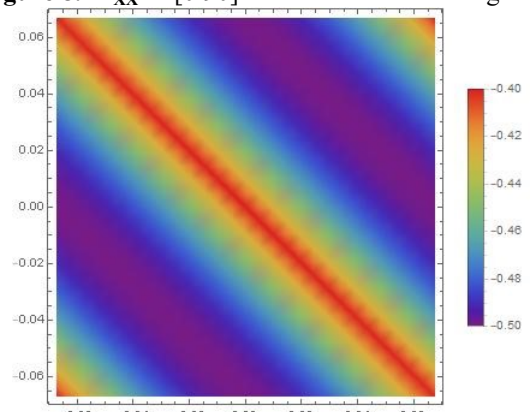


Figure 9. M_{xx} of VS-4 Laminate due to Curing

7. Parametric study using the numerical model

To investigate the effect of changing loading locations of the applied load on snap-through, four selected cases are taken for the study (Figure 10). In all the four cases, loads are applied symmetric to x and y axes of the bistable shell. Initial stage of the parametric study is conducted using both semi-analytical and numerical frameworks. Parametric study in the numerical model can be done with the necessary change in loading points. Whereas in case of the semi-analytical model, work done by external forces (Eq. (13) from section 3) has to be modified with appropriate lever arm distances.

For case-1, Eq.(13) is recalled as:

$$W_F = 4f \left\{ c_{11} \left(\frac{L_x}{2} \right)^2 + c_{12} \left(\frac{L_y}{2} \right)^2 + c_{13} \left(\frac{L_x}{2} \right)^4 + c_{14} \left(\frac{L_y}{2} \right)^4 + c_{15} \left(\frac{L_x}{2} \right)^2 \left(\frac{L_y}{2} \right)^2 \right\}$$

Work done by external forces for the other three cases are modified as follows:

For case-2:

$$W_F = 2f \left\{ c_{11} \left(\frac{L_x}{2} \right)^2 + c_{13} \left(\frac{L_x}{2} \right)^4 \right\} + 2f \left\{ c_{12} \left(\frac{L_y}{2} \right)^2 + c_{14} \left(\frac{L_y}{2} \right)^4 \right\}$$

For case-3:

$$W_F = 4f \left\{ c_{11} \left(\frac{L_x}{4} \right)^2 + c_{12} \left(\frac{L_y}{4} \right)^2 + c_{13} \left(\frac{L_x}{4} \right)^4 + c_{14} \left(\frac{L_y}{4} \right)^4 + c_{15} \left(\frac{L_x}{4} \right)^2 \left(\frac{L_y}{4} \right)^2 \right\}$$

For case-4:

$$W_F = 2f \left\{ c_{11} \left(\frac{L_x}{4} \right)^2 + c_{13} \left(\frac{L_x}{4} \right)^4 \right\} + 2f \left\{ c_{12} \left(\frac{L_y}{4} \right)^2 + c_{14} \left(\frac{L_y}{4} \right)^4 \right\}$$

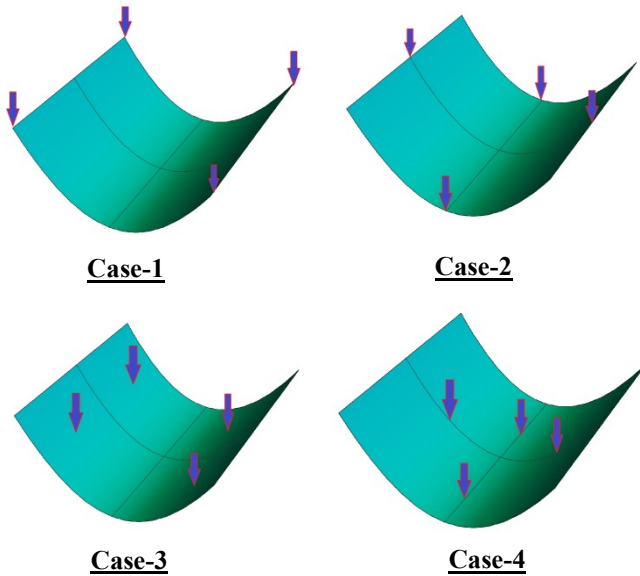


Figure 10. Various load cases considered in the parametric study

Table-5. Snap-through loads for [0/90] laminate

Load case	Snap-through force (N)	
	Semi-Analytical	Numerical
Case-1	3.00	3.17
Case-2	4.42	4.80
Case-3	15.20	16.27
Case-4	24.50	26.51

Table-6. Comparison of snap-through loads for VS

VS family	Snap-through force (N)			
	Case-1	Case-2	Case-3	Case-4
VS-1	1.72	3.07	7.79	23.96
VS-2	2.67	3.55	11.65	22.86
VS-3	3.17	4.80	16.27	26.51
VS-4	3.28	4.85	11.57	26.15
VS-5	2.43	3.88	6.34	11.58

Snap-through loads of [0/90] laminate obtained from the numerical model for all the cases considered are given in Table 5. Lowest snap-through load is observed for case-1 where the loads are applied on all the four corners. As the loading location moves towards the center of laminate, the load requires triggering the snap-through increases. For example, in case-4, the snap-through load is found to be 8.36 times higher than the snap-through load of case-1. From the results, it can be concluded that, applying external loads at corner point can trigger snap-through with lower force requirements (case-1).

To exploit the effect of changing loading positions in VS bistable shells, the proposed numerical parametric study is extended to the selected VS laminates. Comparison of snap-through loads for the selected VS families are given in Table 6. Results shows the similar trend in variation of snap-through loads with loading locations as discussed for [0/90] bistable laminate. For example, in case of VS-2 laminates, snap-through load for case-4 is found to be 8.56 times higher than the load required in case-1. Results from Table 6 also strengthen the potential of using VS laminates in bistable structures, as it is possible to tailor the snap-through loads by altering the fibre orientation appropriately.

Conclusions

In this work, a semi-analytical model is used to predict the bistable shapes of cured VS unsymmetrical laminates and the corresponding snap-through loads. Results from the semi-analytical model are in good agreement with the results from the numerical model. It is also observed that certain VS laminates have a lower requirement of snapping loads than straight fibre laminates, which is of greater advantage in morphing applications. Subsequently, four loading cases are selected for the present study to understand the influence of location of load application during the snap-through process. Lowest snap-through is achieved in case-1. For [0/90] laminate, snap-through loads for case-2 and case-3 are found to be 1.51 and 5.13 times higher than case-1, respectively. The highest snap-through load is observed for case-4 where the snap-through load is

found to be 8.36 times higher than the snap-through load of case-1. Ideal location for the application of transverse loads to trigger the snap-through is at the corner points of bistable shells. It can be concluded that, moving the loads from corner to centre (maintaining same boundary conditions) leads to an increase in snap-through loads.

Disclosures

Free Access to this article is sponsored by SARL ALPHA CRISTO INDUSTRIAL.

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