

Effect of Orientation of Laminates on Bending Behaviour of Delaminated Composite Conoidal Shell Roofs

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Abstract

Literature review shows that research works on the bending behaviour of delaminated composite conoidal shells on rectangular planform are very few in number. In order to fill up this gap a study on static analysis of delaminated composite conoidal shells is done using finite element method. An eight noded curved quadratic isoparametric shell element having five degrees of freedom per node is used for the present work. To ensure the compatibility of deformation and equilibrium of forces and moments at the delamination crack tip a multiple constraint algorithm is developed and incorporated. Results of the present model are validated with help of some benchmark problems available in literature. The study is done to characterize delaminated composite conoids with different types of conventional boundary conditions. Parametric studies are done for angle of orientation, number of layers of laminates and degree of fixity of shells under uniformly distributed loading. Finally after a careful and thorough investigation some pin point conclusions is made as outcome of present study.

Keywords: angle-ply, composite material, conoidal shell, cross-ply, delaminated, finite element method, orientation, uniformly distributed loading

1. Introduction

Thin shell roofs are being widely used in different industrial sectors like civil, aerospace, mechanical and marine engineering. Singly ruled laminated composite conoidal shells are gradually becoming more popular as it can provide large column free space, allow entry of north light, can easily be casted and fabricated. But often these laminated Composite shells undergo a serious problem like delamination. In delamination layers of structure separates out or detached from place to place. However, this problem is not visible from outside and hence behaviour of shell structure needs to be investigated thoroughly in presence of delamination.

Dey et al. [1] used finite element method to study the characteristics of composite paraboloid of revolution shells during bending. Finite element formulation was used by Gim [2] to investigate delaminated double cantilevered plates. He studied on transverse shear deformation of the same. Bolotin [3] investigated on laminated and fiber composites, for delamination and defects like cracks. Parhi et al. [4] investigated laminated composite plates with random delaminations located at multiple places, for the first ply failure. Stegmann [5] studied optimization techniques of composite laminated shell structures consisting of beams and spherical shells, with help of finite element analysis.

The effect of nonlinear terms of the strain measure on the elements has been investigated in this paper. Acharyya et al. [6, 7] used finite element approach for investigating the deflection behaviour of delaminated composite shells. He worked on cylindrical shells with complicated boundary conditions. The damaged conoidal shells were investigated by Kumari et al. [8], focusing on their bending behaviour. In this work graphite epoxy composite material was used for conoidal shells subjected to uniformly distributed loading. Twelve different laminations were considered which include both angle-ply and cross-ply shells. Recent increasing trend of use of thin shell structures for civil constructions were researched by Nikolayevich et al. [9]. They concluded in their paper regarding the chances of the use of composite materials in manufacture of thin shell structures for civil engineering applications and industrial purposes. Ismail et al. [10] worked on damaged composite plate with woven reinforcement. He investigated the buckling characteristics and critical buckling load.

Thorough and extensive review of literature indicates that, the delamination is a common and dangerous problem of composite laminates. Use of different types of composite shell structures is also gradually increasing in various industries.

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Research work on damaged laminated composite plates and shell structures are being addressed by a number of researchers and engineers. However, it has been observed that these works are focusing mainly on cylindrical shells and plates. Hence, this paper aims to report similar studies on industrially important composite conoidal shells.

2. Mathematical Formulation

For conoidal shell analysis an eight noded curved quadratic isoparametric finite element with five degrees of freedom u, v, w, α, β at each node is used. Midplane displacement components u^0, v^0, w^0 are related to the displacement components at any general point at a distance z from the midplane as follows

$$\begin{aligned} u(x, y, z) &= u^0(x, y) - z\alpha(x, y) \\ v(x, y, z) &= v^0(x, y) - z\beta(x, y) \\ w(x, y, z) &= w^0(x, y) = w(x, y) \end{aligned} \quad (1)$$

The linear inplane strains $\epsilon_x, \epsilon_y, \gamma_{xy}$, for undelaminated composite conoidal shells with uniform thickness h , twist radius of curvature R_{xy} and radius of curvature R_y are taken same as of Das et al. [11].

As shown in Fig.1 segments marked as 2, 3 are parts of delaminated shell. In-plane strains, curvatures and transverse shear strains at the crack tip 'o', can be related as shown below (Gim[2]),

$$\{\epsilon^0\}_l = \{\epsilon^0\}_1 + z_l^0 \{k\}_1 \quad (2)$$

Here suffix 1 and l is used to denote undelaminated and delaminated portions marked as 1 and 2, 3 respectively. $\{\epsilon^0\}$ are in-plane strain vectors at the mid-surface, $\{k\}$ are curvature vectors and z_l^0 is the distance, along transverse direction between the mid-surface of undelaminated and delaminated portions respectively.

The global matrices are formed by assembling element stiffness matrix and element load vectors. The relevant boundary conditions are applied on global matrices by deleting the rows and columns, corresponding to zero boundary values. Thus, the general problem of static takes following shape:

$$[K]\{d\} = \{P\} \quad (3)$$

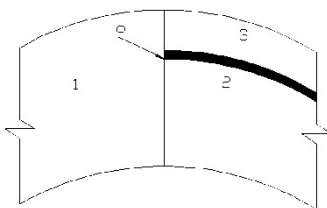


Fig. 1 Details of undelaminated and delaminated shell segments with crack tip

The Gauss elimination technique is used to solve above equation.

Here, $[K]$ is global stiffness matrix, $\{d\}$ and $\{P\}$ are generalised displacement and load vector respectively.

3. Numerical Study

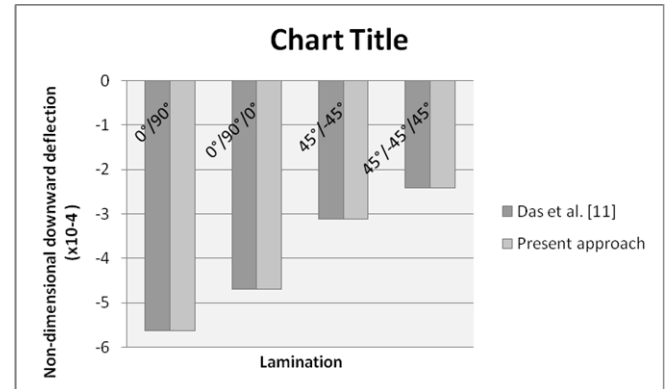
Numerical study of present work is done in two phases. Initially, two validation problems are taken up for checking the correctness of present formulation. Once the exactness of present formulation has been proved, then authors have taken various numerical problems of their own, with some parametric variations for deeper investigation of deflection behavior of damaged conoids.

3.1 Validation Problems

Two validation problems are checked for showing the correctness of present approach. The results obtained from the first validation problem, Das et al. [11], and present formulations are, matching very well. This is the evidence of the exactness of the conoidal shell equations in present mathematical formulation.

First Validation Problem:

Report of maximum non dimensional downward deflection ($\times 10^{-4}$) for simply supported boundary condition



Second Validation Problem:

Again, outputs of present computer code match well with second validation problem results (Table 1), Acharyya et al. [7]. This proves that delaminated composite shell formulation has been rightly done in present mathematical model.

Table 1. Maximum transverse non-dimensional deflections ($\times 10^{-4}$) of delaminated cylindrical shells with corner-supported boundary condition

Lamination	c/a	Acharyya et al. [7]	Present Approach
0°/90°/0°/90°	0.25	59.379	59.378
	0.5	71.736	71.736
45°/-45°/45°/-45°	0.25	26.011	26.010
	0.5	30.371	30.370

Shell features: $h_1=h_2=0.25$, $a=b$, $c=d$, $h=0.01$

Material properties: $G_{12}=G_{13}=0.5E_{22}$, $G_{23}=0.2E_{22}$, $E_{11}=25E_{22}$, $\mu_{12}=0.25$

3.2 Results and discussions

From the results of validation problems it is clear that the present formulation is correct and can be utilized for further investigation of damaged conoidal shells. For deeper analysis of deflection behavior of composite conoidal shells subjected to uniformly distributed loading, following parametric variations have been taken throughout the research work:

- Damaged area represented by c/a is varied from $c/a = 0$ to 0.75 , that is 0% to 56.25% of the total plan area of the shell.
- Two layered and four layered anisotropic laminations are considered, increasing the angle of orientation at step of 15° .
- One case of orthotropic lamination with two and four layers respectively is also taken up.
- Composite material properties and shell features have been taken uniform throughout research work.

These details are:

$$E_{11} = 25E_{22}, G_{12} = G_{13} = 0.5E_{22}, G_{23} = 0.2E_{22}, \nu_{12} = 0.25, h/h = 0.2, a=b, c=d, h=0.01$$

Table 2 contains the results of critical deflections of clamped composite conoids, with uniformly distributed loading. All the conoids have mid plane damaged area at the centre.

Maximum deflection shows increasing trend with increase in extent of delamination that is c/a ratio. This observation stands same for all the boundary conditions. But for some laminations in clamped support like $(-30^\circ/30^\circ)$ and $(0^\circ/90^\circ)$ with $c/a = 0.25$ and 0.75 respectively maximum deflection decreases. Similar trend has been observed in simply supported laminations like $(-45^\circ/45^\circ, 0^\circ/90^\circ)$ with $c/a = 0.5$ and $0.25, 0.5$ respectively). Above observation is also correct for few corner supported lamination cases. In these typical cases, the overall damaged area takes the shape of flat basin like structure, with increase in area of delamination. Thus it can be concluded that, though maximum deflection may experience a decrease but, average deflection of the damaged area always increases. It is evident from results of Table 2 that for clamped boundary condition, maximum deflection increases with increase in angle of orientation. This is true for all the anisotropic shell cases considered here. In case of shells with simply supported edge (Table-3) mixed trend of maximum transverse deflection has been observed. Initially it decreases with increase in orientation angle $(-15^\circ/15^\circ)$ to $(-30^\circ/30^\circ)$ and then it shows increase in deflection value $(-45^\circ/45^\circ)$ to $(-75^\circ/75^\circ)$.

When orientation angle is considered as observation criteria, then corner supported shells (Table-4) become best performer, as in most of cases maximum transverse deflection decreases with increase in orientation angle at a step of 15° . Among all the clamped edge conoidal shells orthotropic laminates show better results as compared to any other anisotropic laminates, if orientation angle is more than $-15^\circ/15^\circ$. However, the above statement is not true for other two edge conditions. The above discussion holds good for all the four layered laminates considered in the present study.

Table 2. Maximum transverse non-dimensional deflections ($\times 10^{-4}$) of conoids with clamped boundary condition

Laminate	Boundary Condition	Clamped			
	Damage	0	0.25	0.5	0.75
$-15^\circ/15^\circ$		0.2513	0.2508	0.2521	0.2623
		[35]	[35]	[35]	[35]
$-15^\circ/15^\circ/15^\circ/-15^\circ$		0.2500	0.2503	0.2530	0.2663
		[36]	[36]	[36]	[36]
$-30^\circ/30^\circ$		0.3724	0.3704	0.378	0.3988
		[48]	[48]	[48]	[48]
$-30^\circ/30^\circ/30^\circ/-30^\circ$		0.3223	0.3257	0.3380	0.3822
		[48]	[48]	[62]	[48]
$-45^\circ/45^\circ$		0.7222	0.7303	0.7463	0.7751
		[61]	[60]	[60]	[61]
$-45^\circ/45^\circ/45^\circ/-45^\circ$		0.5575	0.5669	0.6696	0.7934
		[62]	[62]	[74]	[62]
$-60^\circ/60^\circ$		1.3317	1.3825	1.3990	1.4209
		[74]	[86]	[74]	[74]
$-60^\circ/60^\circ/60^\circ/-60^\circ$		1.0505	1.1212	1.2831	1.4553
		[88]	[88]	[88]	[75]
$-75^\circ/75^\circ$		1.9803	2.0840	2.1728	2.2189
		[87]	[86]	[86]	[87]
$-75^\circ/75^\circ/75^\circ/-75^\circ$		1.6817	1.8718	1.9658	2.0732
		[100]	[100]	[87]	[88]
$0^\circ/90^\circ$		0.3193	0.4035	0.3594	0.2999
		[48]	[100]	[61]	[50]
$0^\circ/90^\circ/90^\circ/0^\circ$		0.3444	0.3474	0.4062	0.6974
		[48]	[48]	[61]	[87]

Note: For all the tables

Bracketed value indicates the position of maximum deflection in each case.

Table 3. Maximum transverse non-dimensional deflections ($\times 10^{-4}$) of conoids with Simply-supported boundary condition

Laminate	Boundary Condition	Simply Supported			
	Damage	0	0.25	0.5	0.75
$-15^\circ/15^\circ$		3.5634	3.62	3.7698	4.3562
		[57]	[57]	[57]	[66]
$-15^\circ/15^\circ/15^\circ/-15^\circ$		2.9871	3.0648	3.2726	4.0826
		[89]	[89]	[90]	[76]
$-30^\circ/30^\circ$		2.8986	2.9012	2.8816	3.5412
		[66]	[66]	[66]	[66]
$-30^\circ/30^\circ/30^\circ/-30^\circ$		1.8913	1.9169	2.2541	3.8221
		[66]	[66]	[76]	[67]
$-45^\circ/45^\circ$		3.1212	3.1499	3.0814	3.5541
		[66]	[66]	[66]	[66]
$-45^\circ/45^\circ/45^\circ/-45^\circ$		2.0302	2.0842	2.3357	4.2265
		[66]	[66]	[66]	[66]
$-60^\circ/60^\circ$		4.1335	4.1426	4.0724	4.7514
		[66]	[66]	[56]	[66]
$-60^\circ/60^\circ/60^\circ/-60^\circ$		3.0344	3.0747	3.3449	4.7919
		[66]	[66]	[76]	[66]
$-75^\circ/75^\circ$		5.7860	5.8242	6.0072	7.4977
		[56]	[56]	[56]	[66]
$-75^\circ/75^\circ/75^\circ/-75^\circ$		4.7401	4.7822	5.0373	6.9351
		[66]	[92]	[66]	[66]
$0^\circ/90^\circ$		5.6292	5.5576	5.5525	6.2751
		[66]	[66]	[66]	[56]
$0^\circ/90^\circ/90^\circ/0^\circ$		4.4940	4.9566	16.1090	5.5930
		[87]	[87]	[113]	[57]

Table 4. Maximum transverse non-dimensional deflections ($\times 10^{-4}$) of conoids with corner-supported boundary condition

Laminate	Boundary Condition	Corner Supported			
	Damage	0	0.25	0.5	0.75
-15°/15°		42.4424	43.7043	47.5574	52.2710
		[70]	[70]	[70]	[70]
-15°/15°/15°/-15°		25.8600	27.7394	33.8294	44.2641
		[78]	[78]	[78]	[78]
-30°/30°		36.9084	37.5192	39.3889	40.9385
		[70]	[70]	[70]	[70]
-30°/30°/30°/-30°		22.0440	23.3398	28.6338	44.8482
		[70]	[79]	[79]	[69]
-45°/45°		30.0701	30.4424	31.9170	32.4614
		[70]	[70]	[70]	[70]
-45°/45°/45°/-45°		20.3729	21.2334	25.3137	32.2900
		[70]	[70]	[79]	[96]
-60°/60°		27.7264	27.9108	28.6349	29.4628
		[53]	[53]	[53]	[53]
-60°/60°/60°/-60°		19.6114	19.7410	21.8359	77.6974
		[70]	[70]	[70]	[102]
-75°/75°		31.1022	31.4340	32.4969	35.6543
		[45]	[45]	[45]	[45]
-75°/75°/75°/-75°		20.6475	20.6894	21.7500	25.4414
		[70]	[70]	[70]	[70]
0°/90°		40.6198	40.6994	40.7756	40.5758
		[70]	[70]	[70]	[78]
0°/90°/90°/0°		22.8742	23.8027	26.9194	32.1934
		[69]	[69]	[78]	[69]

It is very clear from results of all the three boundary conditions that better outcome can be achieved in terms of reduction in maximum transverse deflection value, by increasing the number of layers. However the above fact is not true for few cases of severely damaged shells $c/a=0.75$.

4. Conclusions

Following conclusions have been drawn after extensive analysis of obtained results:

- When boundary conditions are compared keeping rest of the parameters constant, it has been observed that deflection is least for all the cases of clamped boundary condition.
- Among laminates of the clamped group (-15°/15°) performs best.
- For simply supported case, (-30°/30°) stacking sequence is best performer, whereas in corner supported group it is (-60°/60°).
- It has been noticed that as the orientation angle of stacking sequence increases by 15° in each step, transverse deflection also increases in almost all the cases of clamped boundary conditions. But above pattern is reverse for most of the laminates of corner supported group.
- Among simply supported shells, initially maximum transverse deflection decreases with increase in angle of orientation, but later it shows increasing trend with further increase in angle of orientation.

- Transverse deflection reduces in almost all the stacking sequences after increasing the number of laminates.
- It can be concluded that increase in degree of fixity and number of layers, overall helps in decreasing the transverse deflection.

Notations :

E_{11}, E_{22}	Modulus of elasticity
G_{12}, G_{13}, G_{23}	Shear modulus
a, b	Length and width of shell in plan
c, d	Length and width of damaged area in plan
h	Overall shell thickness
q_0	Load intensity of uniformly distributed loading
w	Transverse deflection
u, v, w	Linear displacement along x,y and z axes respectively.
α, β	Rotational displacement along y and x axes respectively
$\epsilon_x, \epsilon_y, \gamma_{xy}$	Inplane strains
ν_{12}	Poisson's ratio
$\bar{w}$	Non-dimensional deflection, given by $(wh^3 E_{22} / q_0 a^4) \times 10^3$

Disclosures

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