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Analytical Solutions for Power Law varied FG laminate Subjected to Thermo-mechanical Loadings

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Abstract

This paper consists of a thermal stress analysis of power law varied functionally graded laminate. In this paper, the exact temperature field obtained by the heat conduction equation, and the same has used for thermal stress analysis. In addition,a simple power law varied temperature field also considered for thermal stress analysis. Heat conductivity and coefficient of thermal expansion have graded as power-law along with the thickness of the domain. The Poisson ratio has kept constant. The numerical solution scheme involved the conversion of the boundary value problem to the initial value problem and the use of the 4th-order Runge Kutta-Grill algorithm for numerical integration. The parametric study includes a thick to thin laminate by considering various aspect ratios.

Keywords: FGM, semi-analytical method, BVP, thermal loading, power law, Heat conduction

1. Introduction

In the laboratory of Japan in 1984, the first functionally graded material (FGM) has invented. The requirement for conflicting material properties has demanded by various industries, and hence the use of pure metal has reduced over time. It has replaced next by alloy then by laminated composites, and both of these have certain drawbacks like melting points are different in alloys and problems of delamination occurs in the laminated composite. So, to overcome such issues, in the thermal area, novel functionally graded (FG) materials have used. FG material composed of metals on one side and ceramic on the other side. The FGM has toughened and strengthened by the metallic composition, while the ceramic in the FGM offers thermal barrier effects and protects the metal from corrosion and oxidation. The composition of metal or ceramic has graded in one or particular direction(s) of FG laminate. In FGM, material variations generally considered to be varied as per exponential law (called E-FGM), power law (called P-FGM), and by both exponential and power law variation (called S-FGM). FGM has wide applications in automotive, aerospace, biomedical, and sports, hence many researchers have attracted more to study various properties and its behavior under different kinds of loadings.

Equivalent single layer (ESL) theories have often used for analyzing the structures. Kiani and Eslami [1] studied the buckling behavior of the FG beam with the help of the Euler Bernoulli (EBT) beam theory under a different type of thermal loading. With the help of first-order shear deformation theory (FOST), Chakraborty et al. [2] developed a new finite beam model for comparison of pure metal behavior with FG beam under static, free vibration and wave propagation problems. Further higher-order shear deformation theory (HOST) has been used by Benatta et al. [3] and presented flexural behavior under three-point loading. Kadoli et al. [4] carried out stress and deflection analyses for various combinations of metal-ceramic, which has achieved by different power law exponent. HOST requires shear correction factors, and to avoid this, a new HOST similar to EBT has developed for dynamic analysis by Thai and Vo [5]. Wattanasakulpong et al. [6] used third-order shear deformation theory to study buckling and vibration of power law varied FG beam. A comparison of all ESL theories has carried out by Ben-Oumrane et al. [7] for S-FGM thick beam subjected to uniformly distributed loads. Apart from ESL theories, many investigators proposed other methods also, like with the help of the FE model axially and transversally loaded FG beam analysis has carried out by Trinh et al. [8]. Similarly, Pietro et al. [9] developed a unified approach for analysis of the FG beam in which materials have either graded linear, parabolic, and cubic variations.

In this paper, an attempt has made to extent semi-analytical formulation developed by Pendhari et al. [10] for thermal stress analysis of FG laminates. The partial differential equation (PDE) of heat conduction has used to obtain the exact variation of temperature field through the thickness of the laminate. This obtained temperature has used to analyze FG laminate by adopting a semi-analytical approach again. For the parametric study, the simple power-law varied temperature has also considered to find out stresses generated in the laminate.

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Developed analytical models include the formation of two-point BVP governed by a group of coupled first-order ODE's (Eqn. 1) within the thickness of the element.

$$\frac{d}{dx_3}y(x_3) = A(x_3)y(x_3) + p(x_3) \qquad (1)$$

Here $y(x_3)$ is an *n*-dimensional vector of primary variables whose number (*n*) equals the order of PDE, for heat conduction formulation, '*n'* is equal to two whereas for stress analysis, is equal to the four. $A(x_3)_{(n,n)}$ Coefficient matrix (a function of material properties in the thickness direction), and $p(x_3)$ is an *n*-dimensional vector of non-homogenous (loading). It has to note that loading terms include only body loads such as inertia loads, thermal loads, electric loads, etc. whereas surface loads have incorporated into the formulation during solution procedure as a boundary condition.

2. Mathematical Formulations

Consider a single layer of thickness '*h*,' FG beam of length '*a*' in 'x_1' direction with finite extent along 'x_2' direction. FG beam has supported at two opposite edges (x_1 =0, *a*) and exposed to the thermal load. This thermal load varies along with the length '*a*' in sinusoidal form.

Under such a situation, laminate is in -stress condition of elasticity in the x_1 - x_3 plane (Fig.1). Coefficient of thermal expansion (α), Elastic modulus (E) and coefficient of thermal conductivity (λ) have varied only through the thickness of laminate accordingly to a power law as,

$$\begin{bmatrix} E(x_3) \\ \alpha(x_3) \\ \lambda(x_3) \end{bmatrix} = \begin{bmatrix} E_b \\ \alpha_b \\ \lambda_b \end{bmatrix} + \left\{ \begin{bmatrix} E_t \\ \alpha_t \\ \lambda_t \end{bmatrix} - \begin{bmatrix} E_b \\ \alpha_b \\ \lambda_b \end{bmatrix} \right\} \left(\frac{x_3}{h} \right)^k \qquad (2)$$

Where, E_b and E_t be Young's modulus of elasticity, α_b and α_t be constant of thermal expansion, λ_b and λ_t are the coefficient of thermal conductivity at the bottom and top surface of the beam, respectively. Further, it has considered that the FG material has uniform properties at every point, and the poison ratio has deemed to be constant through the thickness of the laminate. A thermal load has assumed with only known temperature value at the top and bottom of the laminate surface $\left(T = T_b \text{ at } x_3 = 0 \text{ and } T = T_t \text{ at } x_3 = h\right)$, which has indicated in fig. 1. Detail of heat conduction formulation omitted here to avoid lengthiness of paper. The next paragraph gives the formulation for thermal stress analysis.

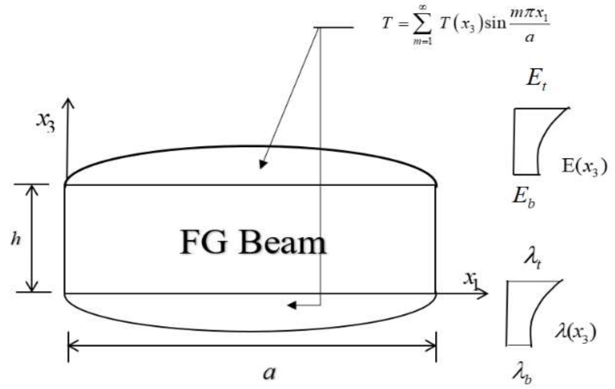

Fig. 1. FG laminate subjected to thermal loading

2.1 2D Stress Analysis Formulation by Semi-Analytical formulation

As per the primary linear theory of elasticity, two-dimensional (2D) strain-displacement relationship, equilibrium equations and constitutive relations in the thermo-elastic environment can be written as,

$$\begin{aligned} \varepsilon_{x_1}(x_1,x_3) &= \frac{\partial u(x_1,x_3)}{\partial x_1} \\ \varepsilon_{x_3}(x_1,x_3) &= \frac{\partial w(x_1,x_3)}{\partial x_3} \\ \gamma_{x_1x_3}(x_1,x_3) &= \frac{\partial u(x_1,x_3)}{\partial x_3} + \frac{\partial w(x_1,x_3)}{\partial x_1} \end{aligned} \qquad (3)$$

$$\begin{aligned} \frac{\partial \sigma_{x_1}(x_1,x_3)}{\partial x_1} + \frac{\partial \tau_{x_1x_3}(x_1,x_3)}{\partial x_3} + B_{x_1} &= 0 \\ \frac{\partial \tau_{x_3x_1}(x_1,x_3)}{\partial x_1} + \frac{\partial \sigma_{x_1x_3}(x_1,x_3)}{\partial x_3} + B_{x_3} &= 0 \end{aligned} \qquad (4)$$

and,

$$\begin{Bmatrix} \sigma_{x_1}(x_1,x_3) \\ \sigma_{x_3}(x_1,x_3) \\ \tau_{x_1x_3}(x_1,x_3) \end{Bmatrix} = \begin{pmatrix} C_{11} & C_{12} & 0 \\ C_{12} & C_{22} & 0 \\ 0 & 0 & C_{33} \end{pmatrix} \begin{Bmatrix} \varepsilon_{x_1}(x_1,x_3) - \alpha(x_3)T \\ \varepsilon_{x_3}(x_1,x_3) - \alpha(x_3)T \\ \gamma_{x_1x_3}(x_1,x_3) \end{Bmatrix} \qquad (5)$$

Here $\alpha(x_3)T$ are the free thermal expansion strain generated due to temperature variation, and to avoid further complications in the analysis, the body forces, B_{x_1}, B_{x_3} per unit volume in x_1 and x_3 directions, respectively, have ignored.

The reduced material coefficients, C_{ij} for plane stress condition, are,

$$C_{11} = \frac{E(z)}{\left(1-\upsilon^2\right)},\ C_{12} = C_{21} = \frac{\upsilon E(z)}{\left(1-\upsilon^2\right)},\ C_{22} = \frac{E(z)}{\left(1-\upsilon^2\right)}$$

The above Eqns. (3), (4) and (5) contains eight unknowns in eight equations which have u, w, $\varepsilon_{x_1}, \varepsilon_{x_3}$, $\sigma_{x_1}, \sigma_{x_3}$ $\gamma_{x_1x_3}$, and $\tau_{x_1x_3}$ After a simple algebraic reduction of the upper sets of equations, a collection of PDEs contains only four dependent variables u, w, σ_{x_3} and $\tau_{x_1x_3}$ received as follows,

$$\frac{\partial u(x_1,x_3)}{\partial x_3}=\frac{\tau_{x_3x_1}(x_1,x_3)}{C_{33}}-\frac{\partial w(x_1,x_3)}{\partial x_1}$$

$$\frac{\partial w(x_1,x_3)}{\partial x_3}=\frac{1}{C_{22}}\begin{bmatrix}\sigma_z(x_1,x_3)-C_{21}\dfrac{\partial u(x_1,x_3)}{\partial x_1}\\ +\alpha(x_3)T(x_1,x_3)(C_{21}+C_{22})\end{bmatrix}$$

$$\begin{aligned}\frac{\partial \tau_{x_3x_1}(x_1,x_3)}{\partial x_3}&=\left[-C_{11}+\left(\frac{C_{12}C_{21}}{C_{22}}\right)\right]\frac{\partial^2 u(x_1,x_3)}{\partial x_1^2}\\&-\frac{C_{12}}{C_{22}}\frac{\partial \sigma_{x_3}(x_1,x_3)}{\partial x_1}\\&-\left[\left(\frac{C_{12}C_{21}}{C_{22}}-C_{11}\right)\alpha(x_3)\right]\frac{\partial T(x_1,x_3)}{\partial x_1}\end{aligned} \quad (6)$$

$$\frac{\partial \sigma_{x_3}(x_1,x_3)}{\partial x_3}=-\frac{\partial \tau_{x_1x_3}(x_1,x_3)}{\partial x_1}$$

With the help of boundary conditions at the support $x=0$ and $x=a$ Fourier trigonometric series expansion, the PDE's given in equation (6) can be converted into coupled first-order ODE as,

$$u(x_1,x_3)=\sum_{m=1}^{\infty}u_m(x_3)\cos\left(\frac{m\pi x_1}{a}\right),$$

$$w(x_1,x_3)=\sum_{m=1}^{\infty}w_m(x_3)\sin\left(\frac{m\pi x_1}{a}\right) \quad (7)$$

and from the fundamental relations of the theory of elasticity, it can be given that,

$$\tau_{x_1x_3}(x_1,x_3)=\sum_{m=1}^{\infty}\tau_{x_1x_3m}(x_3)\cos\frac{m\pi x_1}{a}$$

$$\sigma_{x_3}(x_1,x_3)=\sum_{m=1}^{\infty}\sigma_{x_3m}(x_3)\sin\frac{m\pi x_1}{a} \quad (8)$$

Further, applied transverse loading on the top of the laminate and thermalvariation in the direction x_1 is also express in sinusoidal form as,

$$T(x_1,x_3)=\sum_{m=1}^{\infty}T(x_3)\sin\frac{m\pi x_1}{a} \quad (9)$$

Putting Eqns. (7), (8) and (9), and its differential coefficients into Eqn. (6), ordinary differential equations (ODEs) as mentioned in equation (10) have received,

$$\frac{du_m(x_3)}{dx_3}=\left(-\frac{m\pi}{a}\right)w_m(x_3)+\left(\frac{1}{C_{33}}\right)\tau_{x_1x_3m}(x_3)$$

$$\begin{aligned}\frac{dw_m(x_3)}{dx_3}&=\left(\frac{C_{21}}{C_{22}}\frac{m\pi}{a}\right)u_m(x_3)+\left(\frac{1}{C_{22}}\right)\sigma_{x_3m}(x_3)\\&+\left(\frac{C_{21}+C_{22}}{C_{22}}\right)\alpha(x_3)T(x_3)\end{aligned}$$

$$\begin{aligned}\frac{d\tau_{x_1x_3m}(x_3)}{dx_3}&=\left(\frac{C_{12}C_{21}}{C_{22}}-C_{11}\right)\left(\frac{m^2\pi^2}{a^2}\right)u_m(x_3)\\&-\left(\frac{C_{12}}{C_{22}}\frac{m\pi}{a}\right)\sigma_{x_3m}(x_3)\\&-\left(\frac{C_{12}C_{21}}{C_{22}}-C_{11}\right)\left(\frac{m\pi}{a}\right)\alpha(x_3)T(x_3)\end{aligned}$$

$$\frac{d\sigma_{x_3m}(x_3)}{dx_3}=\left(\frac{m\pi}{l}\right)\tau_{x_1x_3m}(x_3) \quad (10)$$

Eqn. (10) indicate the ruling two-point BVP in ODE's in the domain $0\le x_3\le h$ with stress components known at the upper and downward surfaces of a beam.

3. Numerical Study

A computer program has developed to solve heat conduction formulation as well as stress formulation. We know the temperature at the top is $300^0\,C$ where ceramic has faced, and$20^0\,C$ at the bottom where metal has faced, through thickness temperature distribution has determined with the help of heat conduction solution. Further thermal stress analysis has carried out for exact through thickness distributed temperature (Exact model). A similar exercise has carried out for assumed power law through thickness variation of temperature (PL Model). While performing these exercises, material properties, as mentioned in Table 2, has used.

Different power indices have considered $(k=0)$ for ceramic $(k=10)$ for metals. Based on the convergence studies, around20 to 30 steps have used through the thickness of laminate for numerical integration. Variations in in-plane displacements $(\bar{u}_n)$, transverse displacement $(\bar{w}_n)$,in-plane normal stress $(\bar{\sigma}_x)$, and transverse shear stress $(\bar{\tau}_{xz})$ for different power indices and for aspect ratio five has plotted, which has indicated through fig. 1 to fig. 4, respectively.

Table-2. Material properties

At top,
$z=h\Rightarrow SiC: E=427\ GPa, \lambda=65\ K^{-1}, \alpha=4.3\times10^{-6}\ W_m^{-1}K^{-1}$
At Bottom
$z=0\Rightarrow \text{Al}: E=70\ GPa, \lambda=233\ K^{-1}, \alpha=23.4\times10^{-6}\ W_m^{-1}K^{-1}$

Ref. Ji Ying et al. [11]

From fig.2 and fig.3, it has observed that in-plane and transverse displacementvariations showing nonlinear nature. As the power indices (k) increase from 0 to 10,the difference between the Exact Model and PL Modelresults go on reducing. And $(k=0.8)$ both these variations coincide with each other. For the power index $(k=2)$ and higher, this PL Model results overestimate Exact Model results. Moreover, this observation has partially valid for transverse displacement(fig.3).Fig.4 and Fig.5 shows variations for in-plane normal stress and transverse shear stress, respectively. From fig.4, it has observed that in-plane normal stress followeda parabolic pattern,whereas the cosine and sine curve noticed for transverse shear stress (Fig.5).

Further, it has observed that as the power indices (k) increase from 0 to 10 difference between Exact Model and PL. Model,two results go on reducing, and for $(k=0.8)$ both these variations, coincide with each other. The peak of results of graphs for parabolic as well as sine or cosine curve also shifted to the upper side with increasing power indices. As volume fraction becomes more metallic as power indices increased,the opposite distribution has noticed for both the models as expected, and it hasindicated by $(k=10)$ graphs of fig.4 and 5.

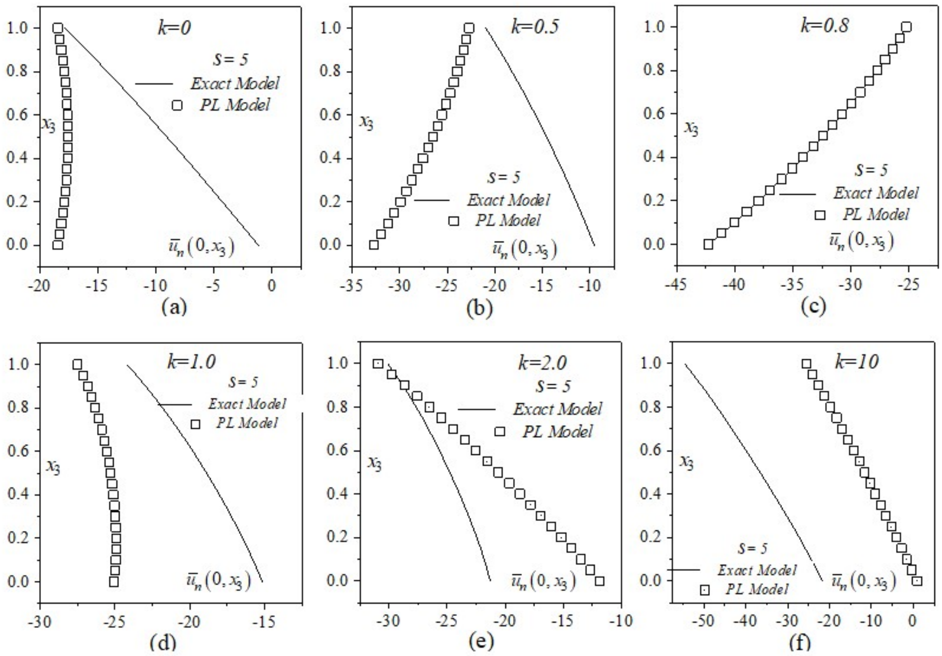

Fig.2Variation of normalized in-plane displacement $\overline{u}_n$ through the thickness of power law varied functionally graded beam under plane-stress condition subjected to thermal load.

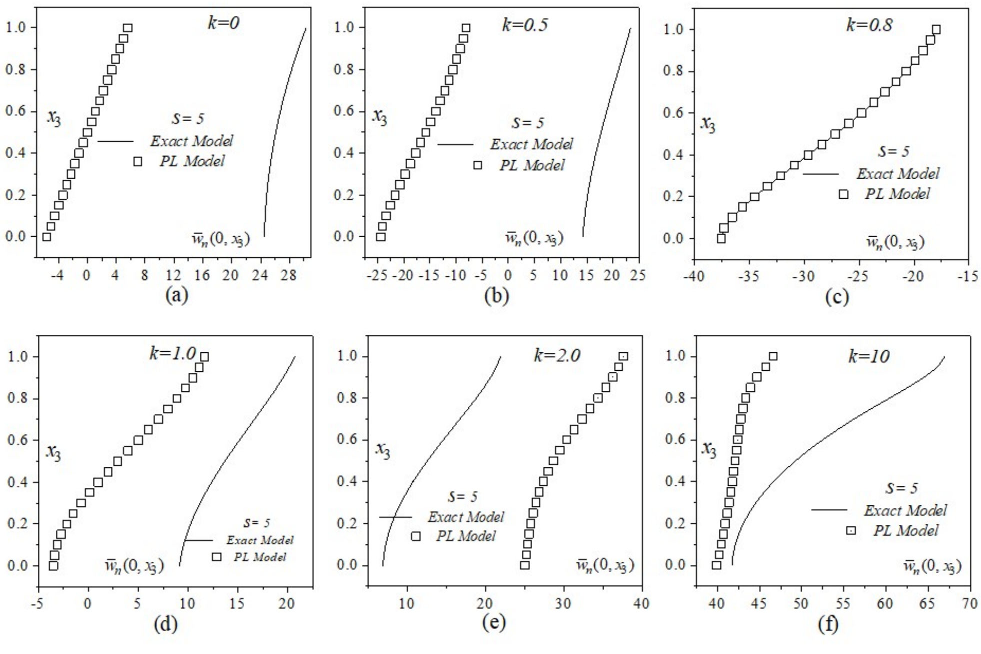

Fig.3Variation of normalized transverse displacement $\overline{w}_n$ through the thickness of power law varied functionally graded beam under plane-stress condition subjected to thermal load.

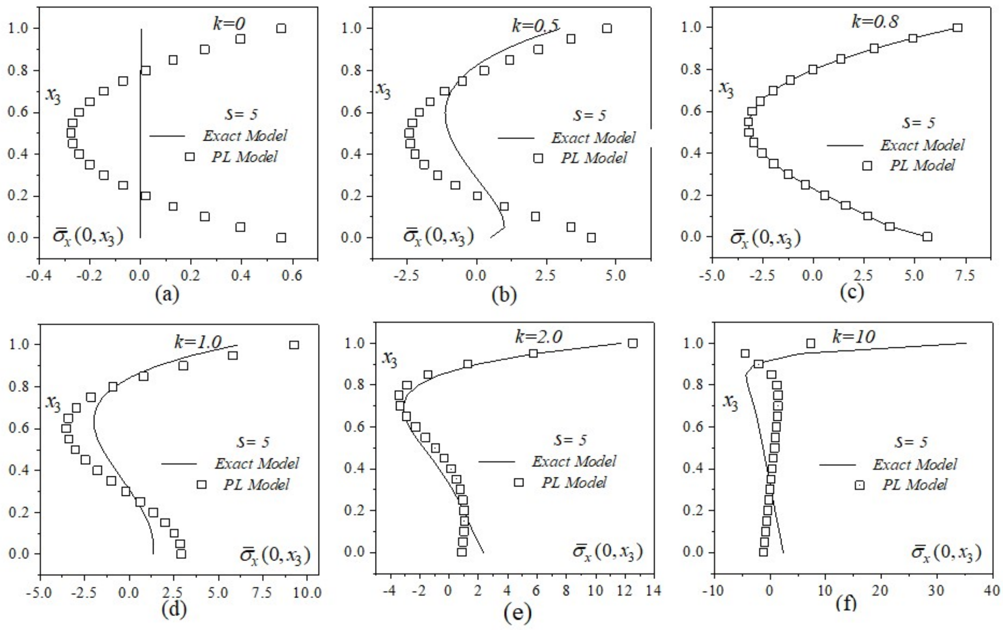

Fig.4Variation of normalized in-plane normal stress $\overline{\sigma}_x$ through thickness of power law varied functionally graded beam under plane-stress condition subjected to thermal load.

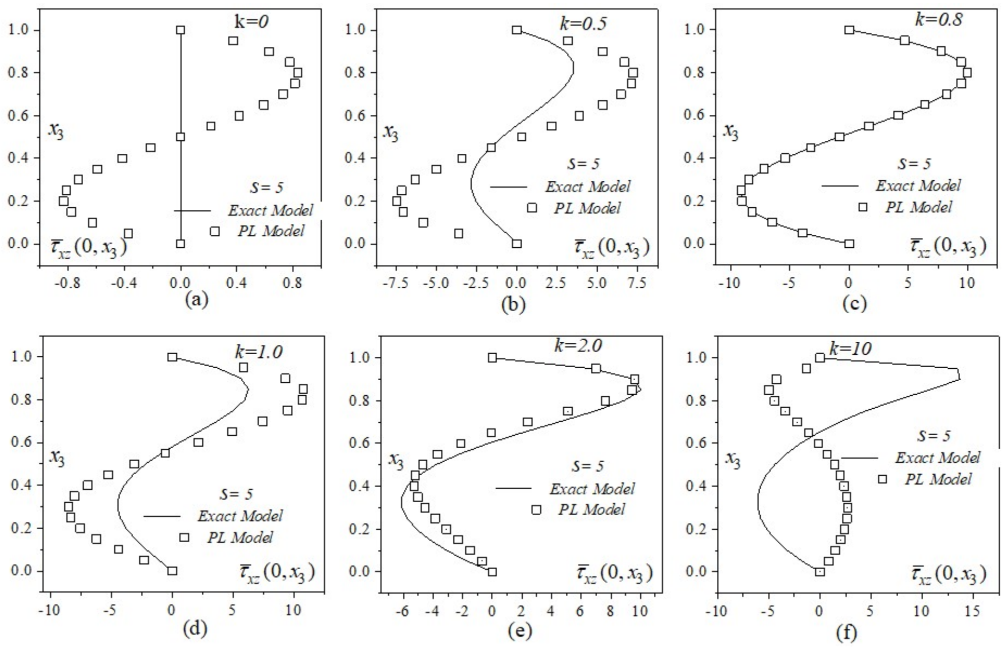

Fig.5 Variation of normalized transverse shear stress $\overline{\tau}_{xz}$ through thickness of power law varied functionally graded beam under plane-stress condition subjected to thermal load.

4. Concluding Remarks

Semi-analytical formulations based on a two-point boundary value problem governed by a set of coupled first-order ordinary differential equations (ODEs) for thermal stress analysis have discussed here. Comparison between power law varied thermal stress and stress generated due to actual temperature distribution, which has gained throughheat conduction formulation for various power indices have carried out. Effect of power indices on displacements and stresses have documented here.Metals are more sensitive for temperatures, as an increase in metallic nature in volume fraction in FG material, increases more fluctuations in stresses.

Disclosures

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