

A step-by-step method for time-dependent analysis of composite beams

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Abstract

Composite beams composed of reinforced concrete slab and steel section joined by mechanical connectors. The concrete slab provides compressive strength, fire resistance, and floor surface, whereas the steel section provides high tensile strength. Steel-concrete composite structures are widely used in frame buildings and bridges due to their economic and structural advantages. In the existing literature, a hybrid analytical-numerical procedure is available for service load analysis of steel-concrete composite beams considering cracking, creep, and shrinkage in concrete. The procedure uses an age-adjusted effective modulus method (AEMM) to model creep and shrinkage effects. The AEMM is computationally efficient however, it compromises accuracy owing to the use of the same aging coefficient for creep and shrinkage. Therefore, in this paper, a step-by-step method (SSM) has been proposed for modeling the creep and shrinkage effects. The method takes into account the progressive concrete cracking with time due to creep and shrinkage. The method is computationally efficient for large composite frame buildings compared to the finite element method since no discretization of the member, along the length and/or across the cross-section, is needed. The method has been validated against experimental results available in the literature. It is observed that the SSM has greater accuracy than the AEMM.

Keywords: AEMM, composite beams, finite element method, SSM

1. Introduction

Steel-concrete composite structures are widely used in frame buildings and bridges due to economic and structural advantages. Composite construction has an added advantage of rapid erection during construction. Composite beams composed of reinforced concrete slab and steel section joined by mechanical connectors. The typical composite beam section is shown in Fig. 1. The concrete slab provides compressive strength, fire resistance, and floor surface, whereas the steel section provides high tensile strength. A monolithic action, with shear connectors between them, enables the concrete slab and steel section to produce a composite beam, making it stiffer and stronger than if the concrete slab and steel section acted alone in non-composite fashion [1].

In sagging or positive bending, the concrete slab is most effective where it is under compression and forms a compressive flange of the composite beam. However, in hogging or negative moment regions, the concrete slab is under tension and may crack at service loads, which implies that the slab is less beneficial since its contribution to strength and stiffness of the composite beam disappears when the concrete cracks [2].

Moreover, there is a progressive cracking of composite beams due to time-dependent effects of creep and shrinkage,

which can result in internal force redistribution and an increase in deflection of beams. In a limit state of design, the structure must be designed to satisfy the requirements of serviceability and strength. Design for strength is relatively straight forward compared to serviceability design. The strength limit state design for composite beams generally applies to the plastification of the steel section in sagging bending or stability consideration in hogging moment regions near the interior supports of the structure [1]. On the other hand, for serviceability limit state design of composite structures, the behavior under sustained service loads is complicated by time-dependent deformation of concrete and increased deflection of beams due to creep and shrinkage. Additional nonlinearity caused by the cracking of concrete over each interior support where the applied tensile stress exceeds the tensile strength of concrete makes further difficulty for long term analysis of composites.

Time analysis of composite structures involves determining strains, stresses, curvatures, and deflections at the critical points and at the critical times during the service life of the structure. The long-term behavior of structures, i.e., final deformation and final internal actions after which effects of creep and shrinkage have taken place, is of most interest in structural design.

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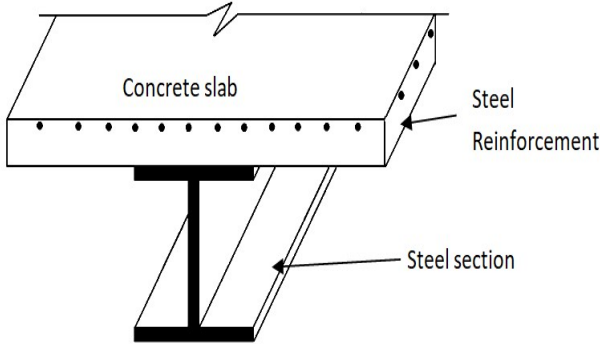


Fig. 1. Typical composite beam section

Several methods have been proposed in the literature for the time-dependent analysis of composite structures in a cracked and uncracked state of concrete in composite beams. Kwak and Seo [3] and Fragiaco et al. [4] developed methods for the time-dependent analysis of continuous composite beams subjected to the ultimate load stage. The beams are divided along the length and across the cross-section to take into account non-linear behavior. The time-dependent analysis of composite structures has also been conducted using numerical procedures under service loads. Inelastic analysis of continuous composite beams using a coupled system of the equation has been proposed by Dezi and Tarantino [5]. The procedure discretizes the beam along the axis and assumes the beam to be uncracked.

Analytical models have also been developed for the time-dependent analysis of two-span continuous composite beams at service loads by Gilbert and Bradford [2]. Analytical expressions for the behavior of two-span composite beams under sustained service loads have been described. Cracking of the slab in the negative moment regions at each interior support has been taken into account.

Further, a hybrid procedure that takes into account the progressive cracking of concrete with time, tension stiffening of cracked concrete, as well as creep and shrinkage effects, has been presented by Chaudhary et al. [6].

In the hybrid procedure [6], close form expressions for flexibility coefficients, end displacements of beam elements, crack lengths, and mid-span deflection of the cracked beam element has been developed. Further, the load vector has been obtained by using closed-form expression. Therefore, the procedure is analytical at the elemental level and numerical at the structural level. Due to closed-form expressions, the hybrid procedure is efficient compared to other methods such as FEM in terms of computational time and effort need for large composite frames. The procedure uses an age-adjusted effective modulus method (AEMM) to model creep and shrinkage effects. The AEMM is computationally efficient; however, it compromises accuracy owing to the use of the same aging coefficient for creep and shrinkage. Therefore, in this paper, a step-by-step method (SSM) has been proposed for modeling the creep and shrinkage effects.

2. Creep of Concrete

For the time-dependent analysis of structures, it is necessary to employ time functions of strain or stress of the materials involved. The strain during the application of the stress (or within seconds after that) is called instantaneous strain. It can be expressed in the form:

$$\varepsilon_e(t_o) = \sigma(t_o) / E_o(t_o) \quad (1)$$

where $\varepsilon_e(t_o)$ is the instantaneous strain, $\sigma(t_o)$ is the applied stress, and $E_o(t_o)$ is the modulus of elasticity of concrete at age t_o , the time of application of stress.

Under sustained stress, the strain increases with time due to the creep of concrete. The strain due to the sustained load after some time t from the application of load is the sum of instantaneous and creep strains and is given by:

$$\varepsilon_c(t_o) = \frac{\sigma_c(t_o)}{E_c(t_o)} [1 + \phi(t, t_o)] \quad (2)$$

where $\phi(t, t_o)$ is a dimensionless creep coefficient, and it is a function of the age of concrete at the time of first loading (t_o) and at the final time (t) at which the strain is calculated.

The creep coefficient ϕ represents the ratio of creep to instantaneous strain; its value increases with the decrease of age at the time of first loading (t_o) and the increase in the length of period of loading ($t - t_o$) during which the stress is sustained [7]. Indian code of practice IS 456 [8] specifies that creep of concrete to be considered as proportional to stress as long as the stress in concrete does not exceed one third of the characteristics compressive strength of concrete. The code also specifies values of creep coefficients in the absence of experimental data for different age of concrete (t_o) first loading. However, in this study, the creep coefficient has been modeled using the creep function given by CEB FIP MODEL CODE 90 [9].

3. Shrinkage of Concrete

Shrinkage of concrete is caused by the drying of concrete in air. When the change in volume caused by shrinkage is restrained, stress develops. In reinforced concrete slab of composite frames and beams, the restraint may be caused by reinforcing steel, supports, or due to the difference in volume change in different parts of the structure.

Stresses caused by the effects of creep and shrinkage must be considered in stress analysis for composite structures [7]. In the absence of experimental data, Indian code IS 456 [8] specifies total shrinkage strain to be taken as 0.0003 for design purposes. In the present study, shrinkage has been modeled using shrinkage law recommended by CEB FIP MODEL CODE 90 [9].

4. Creep Superposition

Within the range of stresses in-service load conditions, superposition of strains due to stress increments or decrements and due to shrinkage can be implemented. When

the magnitude of applied stress changes with time, the total strain of concrete due to the applied stress and shrinkage of concrete is given by Ghali et al. [7]:

$$\varepsilon_c(t) = \sigma_c(t_o) \frac{1 + \phi(t, t_o)}{E_c(t_o)} + \int_0^{\Delta\sigma_c(t)} \frac{1 + \phi(t, \tau)}{E_c(\tau)} d\sigma(\tau) + \varepsilon_{sh}(t, t_o) \quad (3)$$

where t_o is the age of concrete when the initial stress is applied; t is the age of concrete when strain is computed; τ is the intermediate time between t_o and t ; $\sigma(t_o)$ is the initial stress applied at the initial time t_o $d\sigma(\tau)$ is the stress change at the time τ ; $E_c(\tau)$ is the modulus of elasticity of concrete at age τ ; $\phi(t, t_o)$ is the coefficient of creep at time t and loading at the age of concrete t_o ; $\varepsilon_{sh}(t, t_o)$ is the free shrinkage occurring during the time $(t - t_o)$.

5. Step-by-Step Method (SSM)

Under sustained service load where the stress in concrete is unlikely to exceed about half of the compressive strength of concrete, strain due to loading in concrete is proportional to stress, and hence the principle of superposition can be used to estimate deformation caused by a time-varying stress history [10]. Thus, creep strain produced by a stress increment applied at the time τ is assumed to be unaffected by previous stress increment.

According to the principle of superposition, the total strain in concrete at final time t from the time of load application t_o is the sum of elastic instantaneous strain, creep strain, and shrinkage strain, which given by:

$$\varepsilon(t) = \varepsilon_e(t) + \varepsilon_c(t) + \varepsilon_{sh}(t) \quad (4)$$

Upon replacing for each strain components for continuously varying stress history (Fig. 2), the above equation can be written:

$$\varepsilon(t) = \frac{\sigma_o}{E_c(t_o)} [1 + \phi(t, \tau_o)] + \int_0^t \frac{\delta\sigma(\varepsilon)}{\delta\tau} \left[\frac{1 + \phi(t, \tau)}{E_c(\tau)} \right] d\tau + \varepsilon_{sh}(t) \quad (5)$$

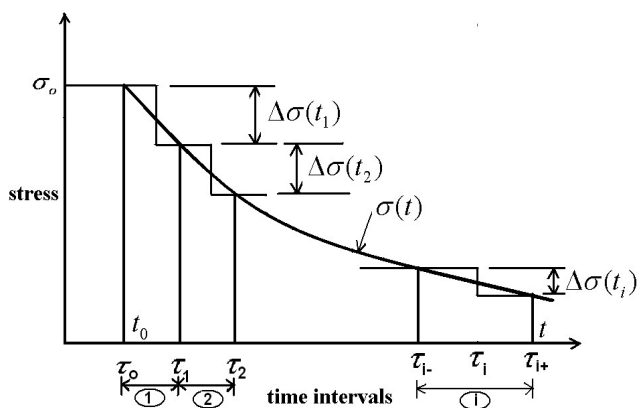


Fig. 2. Time-varying stress history

where σ_o is loading at the initial time t_o ; $E_c(t_o)$ is the modulus of elasticity at the time t_o ; $\phi(t, t_o)$ is creep coefficient; $\delta\sigma(\varepsilon)$ is stress increment for integration; $\delta\tau$ is the time of integration; $\varepsilon_{sh}(t)$ is the shrinkage strain at the time of interest t .

It is observed that Eq. (5) is a superposition equation for a continuously varying stress history shown in Fig. 2. But using the numerical solution for the continuous stress, Eq. (5) can be written as using summation:

$$\varepsilon(t) = \frac{\sigma_o}{E_c(t_o)} + \sum_i \frac{\Delta\sigma(\tau_i)}{E_c(\tau_i)} + \frac{\sigma_o}{E_c(t_o)} \phi(t, \tau_o) + \sum_i \frac{\Delta\sigma(\tau_i) \phi(t, \tau_i)}{E_c(\tau_i)} + \varepsilon_{sh}(t) \quad (6)$$

In the step-by-step (SSM), a numerical solution technique is employed using the incremental form of superposition (Eq. 6). Time is divided into a number of intervals. The greater the number of intervals, the more accurate will be the final prediction of deflection.

The time intervals should be of increasing duration and should be selected so that a similar fraction of the final creep coefficient occurs in each of the time intervals. In a continuously varying stress shown in Fig. 2, the stress increment that occurs in i^{th} interval $\Delta\sigma(\tau_i)$ is assumed to be applied at the end of the interval, i.e., on time τ_i . The increments of instantaneous and creep strain caused by the change in stress $\Delta\sigma(\tau_i)$ is computed at the end of subsequent time intervals using appropriate elastic modulus and creep coefficient $E_c(\tau_i)$ and $\phi(t, \tau_i)$ respectively). The total strain at the end of each time interval is then obtained by superposing the strain increments caused by stress changes in the previous time intervals and due to shrinkage.

Again consider a concrete member subjected to uniaxial stress as shown in Fig. 3 and assume that the magnitude of stress varies with time. At the time, t_o an initial stress value $\sigma_c(t_o)$ is introduced and subsequently increased gradually or stepwise during the time t_o to t . When the variation of stress with time is known, a step-by-step (SSM) can be used to find the strain at any time τ between time t_o to t .

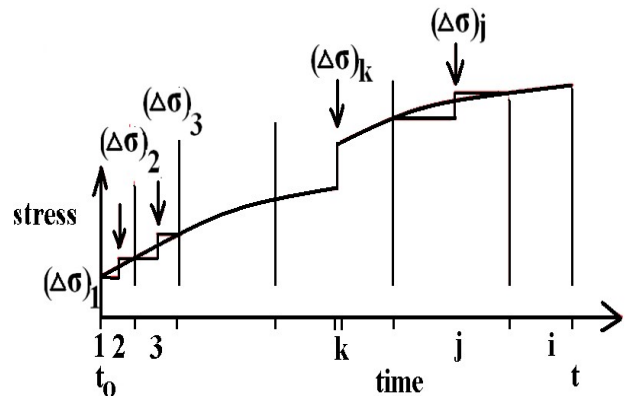


Fig. 3. Step-by-step Method (SSM): division of time and stress for time-dependent analysis

The period of interest for time analysis of structure ($t-t_0$) is divided into a number of intervals, and the stresses are introduced in increment at the end of intervals. For example, the stress $(\Delta\sigma)_j$ shown in Fig. 3 is introduced at the end of j^{th} interval. For a sudden increase in stress, consider an increment introduced at an interval of zero length $[(\Delta\sigma)_k$ in Fig. 3]. The strain at the end of the i^{th} interval can be calculated by using Eq. (3), which can also be written as in the form of Eq. (7) below for a continuously varying stress history [7].

$$\varepsilon(t_{i+}) = \sum_{j=1}^i \left[(\Delta\sigma_c)_j \frac{1+\phi(t_{i+}, t_j)}{E_c(t_j)} \right] + \varepsilon_{sh}(t_{i+}, t_o) \quad (7)$$

where $\varepsilon(t_{i+})$ is the total strain at the end of i^{th} time interval; t_+ is the end of time interval; t_o is the initial time of loading, and the remaining terms are defined in Eq. (5).

The total strain at the end of i^{th} interval can be computed by using the following equation:

$$\begin{aligned} \varepsilon(t_{i+}) = & (\Delta\sigma_c)_i \frac{1+\phi(t_{i+}, t_i)}{E_c(t_i)} \\ & + \sum_{j=1}^{i-1} \left[(\Delta\sigma_c)_j \frac{1+\phi(t_{i+}, t_j)}{E_c(t_j)} \right] + \varepsilon_{sh}(t_{i+}, t_o) \end{aligned} \quad (8)$$

where, σ and ε are the stress and strain respectively with subscripts c referring to concrete; t is the time with subscripts i (j) indicating the end of i^{th} (j^{th}) interval; t_o is the age at the beginning of the period for which the analysis is considered; $\varepsilon_{sh}(t_{i+}, t_o)$ is the shrinkage that would occur if it were free during the period (t_{i+}, t_o) ; $(\Delta\sigma_c)_j$ is change in stress at the end of j^{th} interval.

The change of strain in the i^{th} interval can be computed by taking the difference between the strain values calculated by Eq. (8) at the ends of the $(i-1)^{\text{th}}$ and i^{th} intervals.

$$\begin{aligned} (\Delta\varepsilon_c)_i = & (\Delta\sigma_c)_i \frac{1+\phi(t_{i+}, t_i)}{E_c(t_i)} \\ & + \sum_{j=1}^{i-1} \left[\frac{(\Delta\sigma_c)_j}{E_c(t_j)} [\phi(t_{i+}, t_j) - \phi(t_{i-}, t_j)] \right] + (\Delta\varepsilon_{sh})_i \end{aligned} \quad (9)$$

6. Time-dependent analysis using SSM

As mentioned earlier, the hybrid procedure [6] takes into account the progressive cracking of concrete and time-dependent effects of creep and shrinkage in the composite beam of composite frames and beams under service loads. The cracked span length of the beam element has been modeled as an uncracked zone in the middle and cracked zone, in the end, supports as it is indicated in Fig. 4.

SSM, as discussed in section 5, has been replaced with AEMM in the time-dependent analysis of the hybrid procedure. The procedure is analytical at the elemental level since the various quantities of beam element of the composite frame and beam, such as closed-form expressions for flexibility coefficients, end displacements, crack lengths, and mid-span deflection of the beam have been used.

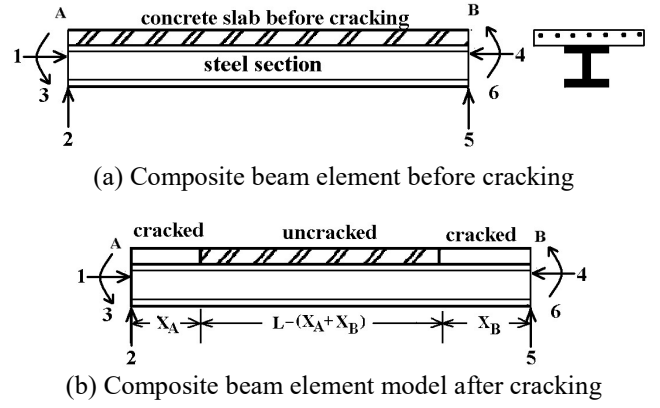


Fig. 4. Cracked span length beam element

Analysis of continuous composite beams under service loads considering the effect of creep and shrinkage phenomena can be done in two parts: instantaneous and time-dependent analysis. Instantaneous analysis needs non-linear analysis, e.g., iteration analysis in which the nonlinearity is introduced by the cracking of concrete at the interior supports of indeterminate structures such as continuous composite beams. Whereas time-dependent analysis of composite structures at service load can be computed using linear elastic analysis for each time interval in which the total time interval is divided into a number of intervals for analysis.

7. Validation

The proposed SSM method for time-dependent analysis of composite beams has been validated with experimental laboratory measurements taken on a full scale continuous composite beam tested over a period of 340 days by Gilbert and Bradford [2].

In a laboratory test, a continuous composite beam was subjected to sustained service load over a period of 340 days. The beam was continuous over 5.8 m. The cross-section and elevation are shown in Fig. 5. The relevant properties and dimensions are: depth of concrete slab, $D_c = 70$ mm; width of the slab, $b = 1000$ mm; depth of steel section, $D_s = 203$ mm; area of steel section, $A_{ss} = 3230$ mm²; moment of inertia of steel section $I_{ss} = 23.6 \times 10^6$ mm⁴; area

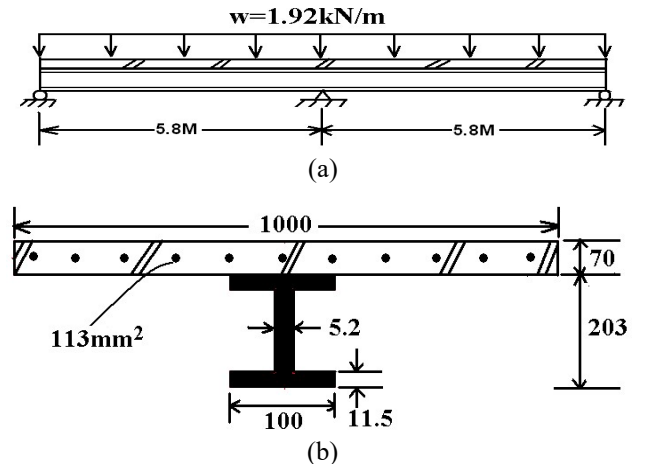


Fig. 5. (a) Elevation and (b) cross-section of two-span continuous

Table-1. Comparison of mid-span deflections

(a)			
Age of loading (days)	Deflection (mm)		Difference (%)
	Experiments	AEMM	
3	5.00	4.08	18.4
7		4.09	18.2
10		4.11	17.8

(b)			
Age of loading (days)	Deflection (mm)		Difference (%)
	Experiments	SSM	
3	5.00	4.84	3.2
7		5.11	-2.2
10		5.24	-4.8

of steel reinforcement for the slab, $A_{sr} = 113 \text{ mm}^2$ and depth of steel reinforcement from top of the slab, $d_{sr} = 15 \text{ mm}$. The properties of concrete at 28 days are: $E_s = 22000 \text{ MPa}$; $\phi = 1.68$; $\varepsilon^{sh} = 0.00052$ and tensile strength $f_t = 2.7 \text{ MPa}$.

The beam was subjected only to its self-weight, i.e., 1.92 kN/m . Time-dependent analysis has been carried out using SSM for modeling of creep and shrinkage effects. Since the authors of the experimental work [2] doesn't specify the age of concrete at the time of first loading, analysis has been carried out for 3, 7, and 10 days of initial loading after casting of the concrete slab. Mid-span deflections from the experiments [2], AEMM [6], and the proposed SSM are compared in Table 1 along with percentage differences. It is seen from Table 1 that the results of the time-dependent analysis using SSM having close agreement than those from AEMM while comparing with the experimental results.

8. Conclusions

It can be concluded that SSM for time-dependent analysis has shown a reasonable accuracy in the computation of deflections of composite beams under sustained service loads. The SSM for modeling of creep and shrinkage has shown close agreement with the experimental

test results available in the literature. Despite the other procedures such as the finite element method (FEM), the proposed procedure is efficient in terms of the computational effort and time needed for computation of time-dependent responses of large composite structures such as the tall composite frame of buildings since no discretization of the member, along the length and/or across the cross-section, is needed. This is due to the use of close form expressions for the structure-property and responses.

Disclosures

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References

- Bradford MA, Gilbert RI. Composite beams with partial interaction under sustained loads. *Journal of Structural Engineering*, 1992; 118(7):1871-1883.
- Gilbert RI, Bradford MA. Time-dependent behavior of continuous composite beams at service loads. *Journal of Structural Engineering*, 1995; 121(2):319-327.
- Kwak HG, Seo YJ. Long-term behavior of composite girder bridges. *Computers & Structures*, 2000; 74(5):583-599.
- Fragiacomo M, Amadio CL, Macorini LO. Finite-element model for collapse and long-term analysis of steel-concrete composite beams. *Journal of Structural Engineering*, 2004; 130(3):489-497.
- Dezi L, Tarantino AM. Creep in composite continuous beams. I: Theoretical treatment. *Journal of Structural Engineering*, 1993; 119(7):2095-2111.
- Chaudhary S, Pendharkar U, Nagpal AK. Hybrid procedure for cracking and time-dependent effects in composite frames at service load. *Journal of Structural Engineering*, 2007; 133(2):166-175.
- Ghali A, Favre R, Elbadry M. *Concrete structures: Stresses and deformations*. Spon Press, 2002; London.
- Bureau of Indian Standard (BIS). Indian standard code for design of plain and reinforced concrete structures. IS 456, 2000; New Delhi.
- Comite' Euro International du Beton-Fe'de'ration Internationale de la Pre'contrainte (CEB-FIP). Model code for concrete structures. CEB-FIP MC 1990; London.
- Gilbert RI. *Time effects in concrete structures*. Elsevier, 1988; Amsterdam.