ASPS Conference Proceedings 1: 121-128 (2022)

Proceedings of

12th Structural Engineering Convention - An International Event (SEC 2022)

Available at https://asps-journals.com/index.php/acp

Prediction of natural frequencies of functionally graded circular and annular plate via differential quadrature method (DQM)

Sumit Khare [a,*], Rahul Vishwakarma [a], Dilsukh Vasara [a] and Rahul Kumar [a]

[a] Department of Mechanical Engineering, Sardar Vallabhbhai National Institute of Technology, Surat-395007, India

Paper ID - 010523

Abstract

In this paper, numerical approach for free vibration response of functionally graded material (FGM) circular and annular plate is examined using differential quadrature method (DQM), and first-order shear deformation theory. The effective material properties through the thickness of the plate are obtained via power-law distribution. The governing partial differential equations (GDEs) are obtained using Hamilton's principle. The five GDEs are discretized via DQM. Several comparison studies were carried out by those published in the literature. The free vibration analysis is investigated to reveal the effects of outer radius to thickness ratio, grading index, radii ratio and boundary conditions.

Keywords: FGM; Circular plate; free vibration; DQM; grading index.

1. Introduction

The pursuit of lightweight yet more robust materials has consistently been a matter of prime concern to engineers worldwide. Advanced composite materials were produced due to the increasing necessity of mixing two or more conventional materials by gradually varying the constituent material's volume fraction. The idea of such a class of material, termed as FGM was proposed by Bever and Duwez, (Bever and Duwez, 1972). FGMs were first exploited by Japanese scientists Koizumi and Niino, (Koizumi and Niino, 1995) (Koizumi, 1997). As the name suggests, functionally graded materials are characterized by its inhomogeneity, such that the gradation of properties of two materials (metal and ceramic usually) occurs according to a predefined function in space from one surface to the other. The metal-rich component is responsible for the toughness of the resultant composite, hence maintaining the structural integrity, and the ceramic rich component attributes to high refractoriness. The continuity of material properties in FGM has significantly mitigated delamination and crackling at the interface problems incurred in conventional laminated composites. FGM structures are extensively utilized as basic structural components in the automotive industry (race car brakes, engine pistons), aeronautical and aerospace industry (rocket nozzles, thermal barrier coatings) electronic appliances (piezoelectric devices, sensors, Integrated circuits), biomaterial electronics and in many structural engineering applications so it is compulsory to investigate their dynamic characteristics. (Alipour and Shariyat, 2014) (Golmakani and Kadkhodayan, 2011) (Jabbari et al., 2014) (Jodaei, Jalal and Yas, 2012).

It is already known that challenges appear while adopting analytical methods in complicated geometries with local elastic supports, patch loading, porosity, mixed boundary conditions, and several other interferences. According to the survey of literature, it is found that many researchers used numerical solution method to analyse free vibration of the FG plates. Efraim and Eisenberger (2007) studied the free vibration behaviour of variable thickness of thick FGM circular plates via the Exact Element Method (EEM). For thin circular FGM plates of variable thickness, Nikkhah-Bahrami and Shamekhi (2008) performed free vibration analysis via the finite element method (FEM). Wirowski (2009) performed free vibration response of thin FGM annular plates via finite difference method (FDM). M and M.m (2010) developed a semi-analytical solution for the free vibration and modal stress analysis of FG circular plates based on the differential transformation method (DTM). Lal and Ahlawat (2015) presented analytical/numerical results for the axisymmetric vibration of FG circular plates subjected to uniform in-plane load using classical plate theory (CPT) and GDEs of the motion solved via a semi-analytical approach. Nie and Zhong (2008) examined the free and forced vibration of FGM annular sectorial plates via a new semi-analytical SSM-DQM approach. In SSM-DQM, state space method (SSM) used to obtain analytical solution

*Corresponding author. Tel: +918813971817; E-mail address: sumitkhare@med.svnit.ac.in

Proceedings of the 12th Structural Engineering Convention (SEC 2022), NCDMM, MNIT Jaipur, India | 19-22 December, 2022

Published online: December 19, 2022

doi:10.38208/acp.v1.482

along the gradation direction and 1-D DQM implemented to approximate solution along the radial direction.
Tornabene (2009) studied the vibration behaviour of moderately thick FG annular plates using FSDT, assuming power-law distribution of material properties and the discretization of the system equations is done by means of the Generalized Differential Quadrature (GDQ) method. Mirtalaie and Hajabasi (2011) used DQM to study the free vibration of FG thin annular sector plates. Yas and Tahouneh, (2012) examined free vibration of elastically supported FGM annular plates via DQM method. Vibration analysis of FG circular plates of variable thickness investigated by Lal and Saini, (2020b) subjected to non-linear temperature distribution using DQM. Alipour et al. (2010) investigated free vibration of two-directional FGM circular plates resting on elastic foundations and subjected to various boundary conditions. Tajeddini et al. (2011) studied three-dimensional free vibration behaviour of FGM circular plates resting on Pasternak foundation using exact elasticity theory and polynomial-Ritz method is used to solve the eigenvalue problem. (Żur, 2018) analysed the behaviour of free vibration of elastically supported FG circular plates based on classical plate theory and to solve the boundary value problem quasi-Green's function was employed. (Lal and Saini, 2020a) studied axisymmetric free vibration of FGM circular plates subjected to a non-linear temperature distribution along the thickness direction via CPT. Molla-Alipour et al. (2020) presented a unified formulation for the free vibration analysis of bidirectional FG annular plates on elastic foundation and the material properties are assumed to vary in both radial and transverse direction by unified formulation. Zhong et al. (2020) investigated the free vibration of multi-directional FG circular plates with variable thickness using isogeometric analysis (IGA).

Kumar et al.(2019) investigate buckling and vibration response of elastically supported FGM plate under porous medium. Roshanbakhsh et al. (2020) presented an exact 3-D solution for free vibration of FG circular plates based on displacement potential functions with surface boundary conditions consisting of a transversely isotropic linearly elastic material. Material properties vary in the thickness direction according to exponential law and power law. Eshraghi and Dag, (2020) analysed forced vibration of FG circular plates based on new domain-boundary element formulation. The resulting set of ordinary differential equations are solved using the Houbolt method. Ahlawat and Lal, (2020) presented the natural frequencies and mode shapes of FG bi-directional circular plates of variable thickness subjected to uniform in-plane peripheral loading and Winkler foundation based on first order shear deformation theory.

Torabi et al. (2013) investigated the free vibration response of a non-uniform cantilever Timoshenko beam using a differential quadrature element method (DQEM). Three-dimensional exact solution provided by Dehghany and Farajpour (2014) for natural frequencies behaviour of simply supported rectangular plate resting on elastic foundation. Jandaghian et al. (2014) provided exact analytical solution for vibration characteristc of simply supported FG circular plate. Vimal et al. (2014) investigated natural frequencies of FG skew plate using FSDT based on FEM. The free vibration responses of laminated composite plates with cutouts are investigated by Bhardwaj et al. (2015) using FEM. Khare and Mittal (2015) analysed free vibration characteristics of the circular and annular plates using FEM. Dehghan et al., (2016) combined finite element and differential quadrature method for 3-D buckling and free vibration analysis of rectangular thick plates partially supported by an elastic foundation. DQM employed by Gupta et al. (2016) for free vibration analysis of inhomogeneous rectangular plate resting on elastic foundation. Torabi and Afshar (2017) presented numerical solution for vibration analysis of cantilevered non-uniform trapezoidal thick plates based on the first shear deformation theory using DQM. The static and dynamic analysis of thin and thick isotropic plates using three dimensional finite element based on the strain approach presented by Lazhar et al. (2021).

In this paper, free vibration response of functionally graded material circular and annular plate is examined using differential quadrature method (DQM), and first-order shear deformation theory. The effective material properties through the thickness of the plate are obtained via power-law distribution. The governing partial differential equations (GDEs) are obtained using Hamilton's principle. The five GDEs are discretized via DQM. Several comparison studies were carried out by those published in the literature. The free vibration analysis is investigated to reveal the effects of outer radius to thickness ratio, grading index, radii ratio and boundary conditions.

The current paper investigates the free vibration of FG-plates circular and annular plate using FSDT, considering clamped and simply supported boundary conditions. The governing equations are discretized using the differential quadrature method, and natural frequencies are obtained for various power-index and boundary conditions. The convergence and comparison study of the present method are examined through the various examples present in the literatures. The effect of various grading index, radii ratio and radius to thickness ratio on free vibration of FGM circular and annular plate are presented.

2. Mathematical Formulation

Consider an FGM circular/annular plate of outer radius "R_o", inner radius "R_i", and thickness "h" with varying properties in the thickness direction which is shown in Fig. 1. The coordinate axis is taken on the middle plane of the plate with z showing the variable across the plate cross section. The effective materials properties of the FGM plate are obtained via Voigt model.

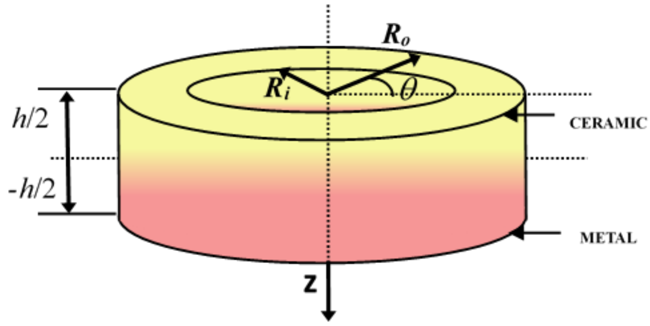

Fig. 1. The geometry of the annular FGM plate

$$P(z)=\left[P_c-P_m\right]\left(\frac{2z+h}{2h}\right)^n+P_m \tag{1}$$

Where, the subscripts c and m refer to ceramics and metals respectively and n is the grading index and P is the material property which includes (young's modulus E, Poisson's ratio μ, and mass density ρ).
The displacements field in space domain is based on FSDT of a point in the circular plate are defined as

$$\begin{Bmatrix} U(r,\theta,z,t) \\ V(r,\theta,z,t) \\ W(r,\theta,z,t) \end{Bmatrix}=\begin{Bmatrix} u(r,\theta,t) \\ v(r,\theta,t) \\ w(r,\theta,t) \end{Bmatrix}+z\begin{Bmatrix} \phi_r(r,\theta,t) \\ \phi_\theta(r,\theta,t) \\ 0 \end{Bmatrix} \tag{2}$$

Where, *u, v* and *w,* represent the unknown displacements of the plate middle surface in the r, θ and z direction, respectively, ϕ_r and ϕ_θ, represent the rotations of rz and θz planes. Besides, t is the time variable. The strains of FGM circular plate can be written as:

$$\begin{Bmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \gamma_{\theta z} \\ \gamma_{rz} \\ \gamma_{r\theta} \end{Bmatrix}=\begin{Bmatrix} \varepsilon_r^0 \\ \varepsilon_\theta^0 \\ \gamma_{\theta z} \\ \gamma_{rz} \\ \gamma_{r\theta}^0 \end{Bmatrix}+z\begin{Bmatrix} \kappa_r^0 \\ \kappa_\theta^0 \\ 0 \\ 0 \\ \kappa_{r\theta}^0 \end{Bmatrix}=\begin{Bmatrix} \frac{\partial u}{\partial r} \\ \frac{1}{r}\left(u+\frac{\partial v}{\partial \theta}\right) \\ \phi_\theta+\frac{1}{r}\frac{\partial w}{\partial \theta} \\ \phi_r+\frac{\partial w}{\partial r} \\ \frac{1}{r}\left(\frac{\partial u}{\partial \theta}-v\right)+\frac{\partial v}{\partial r} \end{Bmatrix}+z\begin{Bmatrix} \frac{\partial \phi_r}{\partial r} \\ \frac{1}{r}\left(\phi_r+\frac{\partial \phi_\theta}{\partial \theta}\right) \\ 0 \\ 0 \\ \frac{1}{r}\left(\frac{\partial \phi_r}{\partial \theta}-\phi_\theta\right)+\frac{\partial \phi_\theta}{\partial r} \end{Bmatrix} \tag{3}$$

Where ε_r^0 and ε_θ^0 are the normal strains; $\gamma_{r\theta}^0$ denote the shear strains; γ_{rz} and $\gamma_{\theta z}$ represent the transverse shear strains; κ_r^0, κ_θ^0 and $\kappa_{r\theta}^0$ express the curvature changes.
The stress-strain relation can be expressed as.

$$\begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_{r\theta} \end{Bmatrix}=\begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}\begin{Bmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \gamma_{r\theta} \end{Bmatrix} \tag{4}$$

$$\begin{Bmatrix} \sigma_{\theta z} \\ \sigma_{rz} \end{Bmatrix}=\begin{bmatrix} Q_{44} & Q_{45} \\ Q_{45} & Q_{55} \end{bmatrix}\begin{Bmatrix} \gamma_{rz} \\ \gamma_{\theta z} \end{Bmatrix} \tag{5}$$

The governing differential equations of the plate are obtained as:

$$\delta u_r:\frac{\partial N_r}{\partial r}+\frac{1}{r}\left(\frac{\partial N_r}{\partial \theta}+N_r-N_\theta\right)=I_0\frac{\partial^2 u}{\partial^2 t}+I_1\frac{\partial^2 \phi_r}{\partial^2 t} \tag{6}$$

$$\delta u_\theta:\frac{\partial N_{r\theta}}{\partial r}+\frac{1}{r}\left(\frac{\partial N_\theta}{\partial \theta}+2N_{r\theta}\right)=I_0\frac{\partial^2 v}{\partial^2 t}+I_1\frac{\partial^2 \phi_\theta}{\partial^2 t} \tag{7}$$

$$\delta u_z:\frac{\partial Q_r}{\partial r}+\frac{1}{r}\left(\frac{\partial Q_\theta}{\partial \theta}+Q_r\right)=I_0\frac{\partial^2 w}{\partial^2 t} \tag{8}$$

$$\partial \phi_r:\frac{1}{r}\left(M_r-M_\theta+\frac{\partial M_{r\theta}}{\partial \theta}\right)+\frac{\partial M_r}{\partial r}-Q_r=I_1\frac{\partial^2 u_0}{\partial^2 t}+I_2\frac{\partial^2 \phi_r}{\partial^2 t} \tag{9}$$

$$\partial \phi_\theta:\frac{1}{r}\left(2M_{r\theta}+\frac{\partial M_\theta}{\partial \theta}\right)+\frac{\partial M_{r\theta}}{\partial r}+-Q_\theta=I_1\frac{\partial^2 v_0}{\partial^2 t}+I_2\frac{\partial^2 \phi_\theta}{\partial^2 t} \tag{10}$$

The forces and moments of a FGM circular plate can be obtained as below

$$\begin{Bmatrix} N_r \\ N_\theta \\ N_{r\theta} \end{Bmatrix}=\begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}\begin{Bmatrix} \varepsilon_r^0 \\ \varepsilon_\theta^0 \\ \gamma_{r\theta} \end{Bmatrix}+\begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix}\begin{Bmatrix} \kappa_r^0 \\ \kappa_\theta^0 \\ \kappa_{r\theta}^0 \end{Bmatrix} \tag{11}$$

R_i

$$\begin{Bmatrix} M_r \\ M_\theta \\ M_{r\theta} \end{Bmatrix}=\begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix}\begin{Bmatrix} \varepsilon_r^0 \\ \varepsilon_\theta^0 \\ \gamma_{r\theta} \end{Bmatrix}+\begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}\begin{Bmatrix} \kappa_r^0 \\ \kappa_\theta^0 \\ \kappa_{r\theta}^0 \end{Bmatrix} \tag{12}$$

$$\begin{Bmatrix} Q_r \\ Q_\theta \end{Bmatrix}=\bar{k}\begin{bmatrix} A_{44} & 0 \\ 0 & A_{55} \end{bmatrix}\begin{Bmatrix} \gamma_{rz} \\ \gamma_{\theta z} \end{Bmatrix} \tag{13}$$

Where, $\bar{k}(=5/6)$ is the shear correction coefficient. The symbol A_{ij}, B_{ij} and D_{ij} are the extensional, coupling and bending stiffness of the plate respectively.

$$A_{ij}=\sum Q_{ij}\left(z_{h/2}-z_{-h/2}\right),\ (i,j=1,2,6) \tag{14}$$

$$A_{ij}=\bar{k}\sum Q_{ij}\left(z_{h/2}-z_{-h/2}\right),\ (i,j=4,5) \tag{15}$$

$$B_{ij}=\frac{1}{2}\sum Q_{ij}\left({z^2}_{h/2}-{z^2}_{-h/2}\right),\ (i,j=1,2,6) \tag{16}$$

$$D_{ij}=\frac{1}{3}\sum Q_{ij}\left({z^3}_{h/2}-{z^3}_{-h/2}\right),\ (i,j=1,2,6) \tag{17}$$

The mass inertias of the plate I_i (i = 0, 1, 2) are defined as follows

$$I_i = \sum \int_{\frac{-h}{2}}^{\frac{h}{2}} \rho\left(1, z, z^2\right) dz \text{ ,} (i = 0,\ 1,\ 2) \qquad (18)$$

3. Method of Solution

In this work, the governing equations are spatially discretized using DQM. The p^{th} spatial derivatives of a generic function $\phi(r)$ at the $(i)^{th}$ point in a one-dimensional region is approximated by (with M divisions in that r-direction), as Khare and Mittal (2019):

$$\left.\frac{\partial^p \varnothing}{\partial r^p}\right|_i = \sum_{k=1}^{M} Q_{rik}^{(p)} \Psi_k \qquad (19)$$

- The weighting coefficient of the value of ϕ at the k^{th} point for the p^{th} order derivative derived with respect to r at the ith point is $Q_{rik}^{(p)}$.
- The value of ϕ at the i^{th} point in the one-dimensional domain is denoted by ψ_k.

$$r_i = R_i + \frac{\left(R_o - R_i\right)}{2}\left\{1 - \cos\left[\frac{\pi\left(i-1\right)}{M-1}\right]\right\}, \qquad (20)$$

$(i = 1, 2, \ldots, M)$

Boundary condition for annular FGM plate can be simulated by controlling translational and rotational degrees of freedom.

$$k_{uj} u + N_r = 0,$$
$$k_{vj} v + N_{r\theta} = 0,$$
$$k_{wj} w + Q_r = 0,$$
$$k_{\phi_r j} \phi_r + M_r = 0,$$
$$k_{\phi_\theta j} \phi_\theta + M_{r\theta} = 0$$

The non-dimensional elastic foundation coefficients are defined as:

$$\overline{k_{i,j}} = \frac{k_{i,j} a^3}{D_{22}} \forall i = u, v, w; j = 1,3$$

$$\overline{k_{i,j}} = \frac{k_{i,j} a}{D_{22}} \forall i = \phi_r, \phi_\theta; j = 1,3$$

$$\overline{k_{i,j}} = \begin{bmatrix} \overline{k_{u,1}} & \overline{k_{u,3}} \\ \overline{k_{v,1}} & \overline{k_{v,3}} \\ \overline{k_{w,1}} & \overline{k_{w,3}} \\ \overline{k_{\phi_r,1}} & \overline{k_{\phi_r,3}} \\ \overline{k_{\phi_\theta,1}} & \overline{k_{\phi_\theta,3}} \end{bmatrix}$$

Thus, considering an annular plate with both edges clamped (CC),

$\overline{k_{i,j}} = [1e12, 1e12, 1e12, 1e12, 1e12; 1e12, 1e12, 1e12, 1e12, 1e12]^T$,

And for an annular plate with both edges simply supported (SS),

$\overline{k_{i,j}} = [1e12, 1e12, 1e12, 0, 1e12; 1e12, 1e12, 1e12, 0, 1e12]^T$

4. Result and Discussions

To establish the accuracy and effectiveness of the employed methodology, a convergence and comparison study is performed. Two types of FGM are considered in the present analysis and depicted in Table 1. The fundamental frequency of a simply supported and clamp supported isotropic circular plate is tabulated in Table 2 and Table 3 respectively. The fundamental frequency parameter $\overline{\omega} = \omega h \sqrt{\rho / E}$ is considered with Ro/h =10. It is evident from the Table 2 and Table 3 that the frequency of all 6 modes is converging at (18x16) and are found to be in good agreement with the solutions by Mohammadi et al. (2013), Nguyen et al. (2015) and Thai et al. (2020). The fundamental frequency of a C-C supported FGM-1 circular plate is plotted in Fig. 2. The effective material properties and plate geometry is taken from Dong, (Dong, 2008). It is observed that the present results in good agreement with the three-dimensional solution using the Chebyshev–Ritz method Dong (2008). Based on the convergence results, it is observed that (18x16) mesh size is sufficient for the predicting free vibration results.

The effects of R_o/h ratio on the fundamental frequency of FGM-2 S-S annular plate is plotted in Fig. 3. It is clearly seen that with increase in R_o/h ratio fundamental frequency increases and the effect of R_o/h ratio is negligible after R_o/h=50.

The effect of grading index on the fundamental natural frequency of FGM-2 S-S annular plate is shown in Fig. 4. It is observed that fundamental frequency decreases with increase in grading index.

The effect of Ri/Ro on the first six frequencies of FGM-2 annular plate with S-S and C-C boundary condition is presented in Table 4 and Table 5, respectively. The geometric parameters (Ro/h=20, n=0.5) are considered for the study. It is observed that with increasing Ri/Ro ratio natural frequency increases. It is notable that fundamental frequency for clamp boundary condition is greater than the simply supported boundary condition but follow the same trend.

Table 1. Functionally graded material properties.

Property	FGM-1		FGM-2	
	Metal (Al)	Ceramic (Al2O3)	Metal (Al)	Ceramic (ZrO2)
E (GPa)	70	380	70	151
v	0.3	0.3	0.3	0.3
ρ(kg/m3)	2707	3800	2707	3000

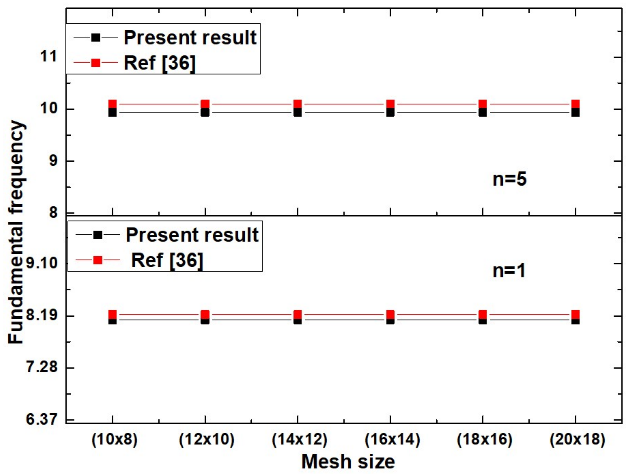

Fig. 2. Convergence and comparison study of C-C FGM-1 annular plate

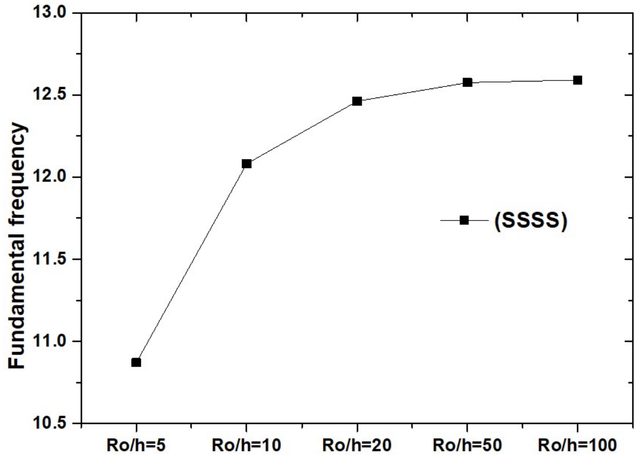

Fig. 3. Influences of R_o/h ratio on fundamental frequency of FGM-2 S-S circular plate with different porosity index (n=1, R_o=10, R_i=1)

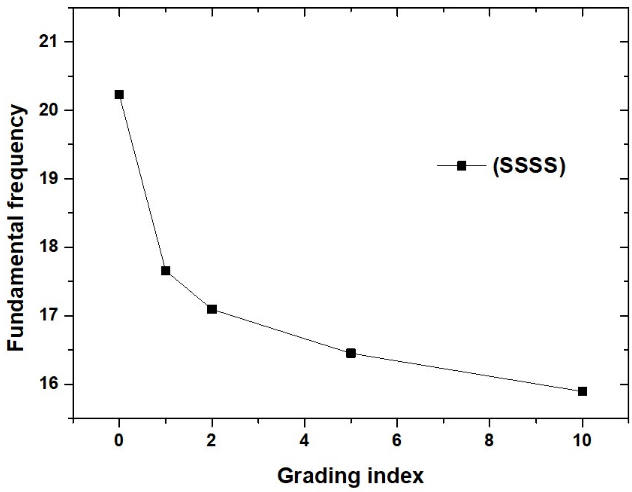

Fig. 4. Influences of grading index on fundamental frequency of FGM-2 S-S circular plate with different porosity index (R_o=10, R_i = 3, R_o/h=10)

Table 2. Convergence and validation studies of fundamental frequency of S-S isotropic circular plate

Methods	Modes					
	I	II	III	IV	V	VI
Present (10x8)	4.8926	13.8548	25.4288	29.9457	49.2279	69.9271
Present (12x10)	4.9002	13.7605	25.6591	29.8811	41.3931	49.2661
Present (14x12)	4.9058	13.7124	25.4949	29.841	40.4513	49.3601
Present (16x14)	4.9101	13.6541	25.5612	29.8145	39.8385	49.4721
Present (18x16)	4.9135	13.5868	25.5688	29.7956	39.9384	49.6281
Present (20x18)	4.9162	13.495	25.5772	29.7813	39.9238	49.8776
Mohammadi et al. (2013)	4.9345	13.898	25.613	29.72	39.957	48.479
Nguyen et al. (2015)	4.9304	13.86	25.482	29.548	39.656	48.046
Nguyen et al. (2015)	4.9304	13.859	25.48	29.539	39.633	48.005
Thai et al. (2020)	4.9304	13.847	25.441	29.57	39.601	48.197

Table 3. Convergence and validation studies of fundamental frequency of C-C isotropic circular plate.

Methods	Modes					
	I	II	III	IV	V	VI
Present (10x8)	10.2421	21.5615	34.7709	39.8025	61.2187	84.1527
Present (12x10)	10.2302	21.5224	34.9939	39.7729	52.7139	61.1506
Present (14x12)	10.2243	21.5355	34.7868	39.7585	51.5813	61.1721
Present (16x14)	10.2210	21.5477	34.8487	39.7507	50.8645	61.199
Present (18x16)	10.2189	21.5733	34.8467	39.746	50.9775	61.2455
Present (20x18)	10.2176	21.6231	34.847	39.743	50.961	61.3302
Mohammadi et al. (2013)	10.216	21.26	34.877	39.771	51.03	60.829
Nguyen et al. (2015)	10.185	21.148	34.613	39.367	50.495	60.054
Nguyen et al. (2015)	10.184	21.143	34.589	39.362	50.439	59.958
Thai et al. (2020)	10.184	21.136	34.558	39.443	50.563	60.408

Table 4. Influences of R_i/R_o ratio on fundamental frequency of S-S FGM circular plate with different annularity ratio.

Modes	R_i/R_o ratio					
	0	0.1	0.2	0.3	0.4	0.5
I	4.4657	12.9548	15.0487	18.8917	25.1402	35.6234
II	12.3506	14.9573	17.18	20.8362	26.8546	37.1204
III	22.7522	23.0278	24.1988	26.8943	32.0825	41.6426
IV	26.3504	35.2932	35.5874	36.9969	40.8508	49.214
V	35.3051	45.3632	49.7517	50.3104	52.7787	59.6203
VI	42.531	49.4006	50.1058	50.6515	53.0902	59.8941

Table 5. Influences of R_i/R_o ratio on fundamental frequency of C-C FGM circular plate with different annularity ratio.

Modes	R_i/R_o ratio					
	0	0.1	0.2	0.3	0.4	0.5
I	9.1074	23.8442	30.1402	39.2044	52.8686	74.8135
II	18.7729	25.1632	31.3717	40.2659	53.7693	75.5702
III	30.6618	31.9081	36.2441	44.0163	56.7548	77.982
IV	34.91	44.6118	46.2352	51.5578	62.4966	82.4225
V	44.5345	60.0887	60.467	63.2251	71.502	89.2511
VI	52.759	60.4674	60.8328	63.542	71.7525	89.4413

5. Conclusions

This study uses first order shear deformation theory to examine the free vibration analysis of FGM circular and annular plate. The effective material properties of FGM circular and annular plate along the thickness direction are calculated using power law. DQM method has been used to examine the results for simply supported and clamped boundary conditions. The convergence and comparison study of the present method are examined through the various examples present in the literatures. The effect of various grading index, radii ratio and radius to thickness ratio on free vibration of FGM circular and angular plate were studied and the results of this examination can be summarized as:

- The present solution methodology is accurate and fast convergence rate
- The increasing of the grading index results in the decrease of fundamental frequency.
- The increasing of the radii ratio results in the rise of fundamental frequency.
- The increasing of the radius to thickness ratio results in the rise of fundamental frequency.
- The fundamental frequency for clamp boundary condition is greater than the simply supported boundary condition.

The results introduced in this paper can fill in as benchmark solutions for future examination plate structure.

Disclosures

Free Access to this article is sponsored by SARL ALPHA CRISTO INDUSTRIAL.

References

[1]. Ahlawat, N. and Lal, R. (2020) ‘Effect of Winkler Foundation on Radially Symmetric Vibrations of Bi-Directional FGM Non-Uniform Mindlin’s Circular Plate Subjected to In-Plane Peripheral Loading’, *Journal of Solid Mechanics*, 12(2), pp. 455–475. doi:10.22034/jsm.2019.1873720.1466.

[2]. Alipour, M.M. and Shariyat, M. (2014) ‘Analytical stress analysis of annular FGM sandwich plates with non-uniform shear and normal tractions, employing a zigzag-elasticity plate theory’, *Aerospace Science and Technology*, 32(1), pp. 235–259. doi:10.1016/j.ast.2013.10.007.

[3]. Alipour, M.M., Shariyat, M. and Shaban, M. (2010) ‘A semi-analytical solution for free vibration of variable

thickness two-directional-functionally graded plates on elastic foundations', *International Journal of Mechanics and Materials in Design*, 6(4), pp. 293–304. doi:10.1007/s10999-010-9134-2.

[4]. Bever, M.B. and Duwez, P.E. (1972) 'Gradients in composite materials', *Materials Science and Engineering*, 10, pp. 1–8. doi:10.1016/0025-5416(72)90059-6.

Bhardwaj, H., Vimal, J. and Sharma, A. (2015) 'Study of free vibration analysis of laminated composite plates with triangular cutouts', *Engineering Solid Mechanics*, 3(1), pp. 43–50.

[5]. Dehghan, M., Nejad, M. and Moosaie, A. (2016) 'An effective combination of finite element and differential quadrature method for analyzing of plates partially resting on elastic foundation', *Engineering Solid Mechanics*, 4(4), pp. 201–218.

[6]. Dehghany, M. and Farajpour, A. (2014) 'Free vibration of simply supported rectangular plates on Pasternak foundation: An exact and three-dimensional solution', *Engineering Solid Mechanics*, 2(1), pp. 29–42.

[7]. Dong, C.Y. (2008) 'Three-dimensional free vibration analysis of functionally graded annular plates using the Chebyshev–Ritz method', *Materials & Design*, 29(8), pp. 1518–1525. doi:10.1016/j.matdes.2008.03.001.

[8]. Efraim, E. and Eisenberger, M. (2007) 'Exact vibration analysis of variable thickness thick annular isotropic and FGM plates', *Journal of Sound and Vibration*, 299(4), pp. 720–738. doi:10.1016/j.jsv.2006.06.068.

[9]. Eshraghi, I. and Dag, S. (2020) 'Forced vibrations of functionally graded annular and circular plates by domain-boundary element method', *ZAMM - Journal of Applied Mathematics and Mechanics / Zeitschrift für Angewandte Mathematik und Mechanik*, 100(8), p. e201900048. doi:10.1002/zamm.201900048.

[10]. Golmakani, M.E. and Kadkhodayan, M. (2011) 'Large deflection analysis of circular and annular FGM plates under thermo-mechanical loadings with temperature-dependent properties', *Composites Part B: Engineering*, 42(4), pp. 614–625. doi:10.1016/j.compositesb.2011.02.018.

[11]. Gupta, U., Sharma, S. and Singhal, P. (2016) 'DQM modeling of rectangular plate resting on two parameter foundation', *Engineering Solid Mechanics*, 4(1), pp. 33–44.

[12]. Jabbari, M. *et al.* (2014) 'Buckling Analysis of a Functionally Graded Thin Circular Plate Made of Saturated Porous Materials', *Journal of Engineering Mechanics*, 140(2), pp. 287–295. doi:10.1061/(ASCE)EM.1943-7889.0000663.

[13]. Jandaghian, A., Jafari, A. and Rahmani, O. (2014) 'Vibrational response of functionally graded circular plate integrated with piezoelectric layers: An exact solution', *Engineering Solid Mechanics*, 2(2), pp. 119–130.

[14]. Jodaei, A., Jalal, M. and Yas, M.H. (2012) 'Free vibration analysis of functionally graded annular plates by state-space based differential quadrature method and comparative modeling by ANN', *Composites Part B: Engineering*, 43(2), pp. 340–353. doi:10.1016/j.compositesb.2011.08.052.

[15]. Khare, S. and Mittal, N. (2015) 'Free vibration analysis of thin circular and annular plate with general boundary conditions', *Engineering Solid Mechanics*, 3(4), pp. 245–252.

[16]. Khare, S. and Mittal, N.D. (2019) 'Axisymmetric bending and free vibration of symmetrically laminated circular and annular plates having elastic edge constraints', *Ain Shams Engineering Journal*, 10(2), pp. 343–352. doi:10.1016/j.asej.2018.10.006.

[17]. Koizumi, M. (1997) 'FGM activities in Japan', *Composites Part B: Engineering*, 28(1), pp. 1–4. doi:10.1016/S1359-8368(96)00016-9.

[18]. Koizumi, M. and Niino, M. (1995) 'Overview of FGM Research in Japan', *MRS Bulletin*, 20(1), pp. 19–21. doi:10.1557/S0883769400048867.

[19]. Kumar, R. *et al.* (2019) 'Meshfree approach on buckling and free vibration analysis of porous FGM plate with proposed IHHSDT resting on the foundation', *Curved and Layered Structures*, 6(1), pp. 192–211. doi:10.1515/cls-2019-0017.

[20]. Lal, R. and Ahlawat, N. (2015) 'Axisymmetric vibrations and buckling analysis of functionally graded circular plates via differential transform method', *European Journal of Mechanics - A/Solids*, 52, pp. 85–94. doi:10.1016/j.euromechsol.2015.02.004.

[21]. Lal, R. and Saini, R. (2020a) 'Vibration analysis of FGM circular plates under non-linear temperature variation using generalized differential quadrature rule', *Applied Acoustics*, 158, p. 107027. doi:10.1016/j.apacoust.2019.107027.

[22]. Lal, R. and Saini, R. (2020b) 'Vibration analysis of functionally graded circular plates of variable thickness under thermal environment by generalized differential quadrature method', *Journal of Vibration and Control*, 26(1–2), pp. 73–87. doi:10.1177/1077546319876389.

[23]. Lazhar, D. *et al.* (2021) 'Solid strain based finite element implemented in ABAQUS for static and dynamic plate analysis', *Engineering Solid Mechanics*, 9(4), pp. 449–460.

[24]. M, S. and M.m, A. (2010) 'A DIFFERENTIAL TRANSFORM APPROACH FOR MODAL ANALYSIS OF VARIABLE THICKNESS TWO-DIRECTIONAL FGM CIRCULAR PLATES ON ELASTIC FOUNDATIONS', 11(215), pp. 15–38.

[25]. Mirtalaie, S.H. and Hajabasi, M.A. (2011) 'Free vibration analysis of functionally graded thin annular sector plates using the differential quadrature method', *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, 225(3), pp. 568–583. doi:10.1243/09544062JMES2232.

[26]. Mohammadi, M., Ghayour, M. and Farajpour, A. (2013) 'Free transverse vibration analysis of circular and annular graphene sheets with various boundary conditions using the nonlocal continuum plate model', *Composites Part B: Engineering*, 45(1), pp. 32–42. doi:10.1016/j.compositesb.2012.09.011.

[27]. Molla-Alipour, M., Shariyat, M. and Shaban, M. (2020) 'Free Vibration Analysis of Bidirectional Functionally Graded Conical/Cylindrical Shells and Annular Plates on Nonlinear Elastic Foundations, Based on a Unified Differential Transform Analytical Formulation', *Journal of Solid Mechanics*, 12(2), pp. 385–400. doi:10.22034/jsm.2019.1869981.1450.

[28]. Nguyen, N.-T. *et al.* (2015) 'An efficient computational approach for size-dependent analysis of functionally graded nanoplates', *Computer Methods in Applied Mechanics and Engineering*, 297, pp. 191–218. doi:10.1016/j.cma.2015.07.021.

[29]. Nie, G.J. and Zhong, Z. (2008) 'Vibration analysis of functionally graded annular sectorial plates with simply supported radial edges', *Composite Structures*, 84(2), pp. 167–176. doi:10.1016/j.compstruct.2007.07.003.

[30]. Nikkhah-Bahrami, M. and Shamekhi, A. (2008) 'Free Vibration Analysis of Circular FGM Plate Having Variable Thickness Under Axisymmetric Condition by Finite Element Method', in. *ASME 2005 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, American Society of Mechanical Engineers Digital Collection, pp. 1963–1969. doi:10.1115/DETC2005-84021.

[31]. Roshanbakhsh, M.Z., Tavakkoli, S.M. and Navayi Neya, B. (2020) 'Free vibration of functionally graded thick circular plates: An exact and three-dimensional solution', *International Journal of Mechanical Sciences*, 188, p. 105967. doi:10.1016/j.ijmecsci.2020.105967.

[32]. Tajeddini, V., Ohadi, A. and Sadighi, M. (2011) 'Three-dimensional free vibration of variable thickness thick circular and annular isotropic and functionally graded plates on Pasternak foundation', *International Journal of Mechanical Sciences*, 53(4), pp. 300–308. doi:10.1016/j.ijmecsci.2011.01.011.

[33]. Thai, C.H., Ferreira, A.J.M. and Phung-Van, P. (2020) 'A nonlocal strain gradient isogeometric model for free vibration and bending analyses of functionally graded plates', *Composite Structures*, 251, p. 112634. doi:10.1016/j.compstruct.2020.112634.

[34]. Torabi, K. and Afshari, H. (2017) 'Vibration analysis of a cantilevered trapezoidal moderately thick plate with variable thickness', *Engineering Solid Mechanics*, 5(1), pp. 71–92.

[35]. Torabi, K., Afshari, H. and Heidari-Rarani, M. (2013) 'Free vibration analysis of a non-uniform cantilever Timoshenko beam with multiple concentrated masses using DQEM', *Engineering Solid Mechanics*, 1(1), pp. 9–20.

[36]. Tornabene, F. (2009) 'Free vibration analysis of functionally graded conical, cylindrical shell and annular plate structures with a four-parameter power-law distribution', *Computer Methods in Applied Mechanics and Engineering*, 198(37), pp. 2911–2935. doi:10.1016/j.cma.2009.04.011.

[37]. Vimal, J. *et al.* (2014) 'Free vibration analysis of moderately thick functionally graded skew plates', *Engineering Solid Mechanics*, 2(3), pp. 229–238.

[38]. Wirowski, A. (2009) 'Free vibrations of thin annular plates made from functionally graded material', *PAMM*, 9(1), pp. 261–262. doi:10.1002/pamm.200910104.

[39]. Yas, M.H. and Tahouneh, V. (2012) '3-D Free vibration analysis of thick functionally graded annular plates on Pasternak elastic foundation via differential quadrature method (DQM)', *Acta Mechanica*, 223(1), pp. 43–62. doi:10.1007/s00707-011-0543-6.

[40]. Zhong, S. *et al.* (2020) 'Isogeometric vibration analysis of multi-directional functionally gradient circular, elliptical and sector plates with variable thickness', *Composite Structures*, 250, p. 112470. doi:10.1016/j.compstruct.2020.112470.

[41]. Żur, K.K. (2018) 'Quasi-Green's function approach to free vibration analysis of elastically supported functionally graded circular plates', *Composite Structures*, 183, pp. 600–610. doi:10.1016/j.compstruct.2017.07.012.