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Comparative study on response of jacket platforms under regular waves using Airy's and Stokes' fifth-order wave theories

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Abstract

Despite the random nature of ocean waves, to provide engineering solutions, regular wave theories are commonly used to estimate wave forces from the Morison equation in offshore structural analysis. These wave theories can be defined by three basic parameters, namely water depth, wave height and wave period. The applicability of a particular wave theory to a given set of wave characteristics and water depth is governed chiefly by the ratios of water depth and wave height to the wavelength. Airy's linear wave theory, on account of its simplicity, is a popular choice, especially for preliminary calculations and for providing insight into the basic characteristics of wave-induced water motion. It is, however, applicable to small wave heights and in many conditions the linear theory is incapable of providing a satisfactory assessment of the water particle kinematics. A nonlinear theory is then required, and Stokes' fifth order wave theory, based on the expansion of the wave solution in series form, provides a more accurate representation of the free-surface and is generally used for high waves in deep water. In this paper, a comparative study is conducted between the wave forces obtained from Morison equation utilising Airy's wave theory and Stokes' fifth order wave theory, acting at different levels of a jacket platform. Two sets of water depth, wave height and wave period are selected so that they satisfy the region of applicability of Airy's wave theory and Stokes' fifth order wave theory respectively. Two example four-legged jacket platforms are considered. It is found that for design purposes, in most cases the wave forces and structural responses from Airy's wave theory are more conservative as compared to those from Stokes' fifth order wave theory, though the converse is obtained in case of deep water conditions near the seabed. Overall, in deep water, high wave conditions, apart from Stokes' fifth order wave theory, Airy's wave theory may be used for preliminary estimations of wave forces and deck displacements, but in shallow water conditions, Stokes' fifth order wave theory is entirely invalid.

Keywords: Jacket platform, Airy's wave theory, Morison equation, Wave force, Stokes' fifth order wave theory, Dynamic analysis

1. Introduction

Offshore and marine structures are constructed for a variety of purpose, such as for the exploration of oil and natural gas, harnessing power from the sea, oceanographic research, space operation, fish-farming activities, recreation, etc., and thereby, their sustained good structural performance has significant contribution to the country's economy. The construction of these structures requires a detailed knowledge of the ocean environment for analysing the forces acting on them. Apart from the permanent (dead) and the operating live loads, these structures are exposed to different environmental loads, as from wind, earthquake, wave, current, tide, ice, marine growth and seabed movements, that govern their design, of which the most vital is the wave loading.

The modelling of ocean waves may be carried out by two different approaches. The first approach is that of the single wave method, where regular wave theories are applicable. The other would be the irregular wave approach, in which wave spectrums are used to characterise random waves. In the single wave approach method, regular waves are characterised by three parameters, namely, the water depth (d), wave height (H) and wave period (T). Various wave theories present in literature include Airy's wave theory [1], the second order and fifth order Stokes' wave theory [2], the cnoidal wave theory [3], the numerical theory by Chappelear [4], the solitary wave theory [5] and the stream function theory [6]. Airy's wave theory is the simplified linear wave theory which is valid when the wave height is much lower as compared to the wavelength and the water depth. This wave theory is applicable at any water depth so long as the wave height is small. Airy's waves are sinusoidal and produce equal crest and trough. Stokes' fifth order wave theory is the most commonly used non-linear wave theory applicable for high waves in deep and intermediate water depths. These waves have a steep crest and flat trough. The particle orbit for non-linear waves in deep water is circular but becomes highly elliptical in shallow water. Thus, Stokes' wave theory provides inaccurate results in shallow water depth. Cnoidal wave theory is a non-linear wave theory applicable in shallow

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water depth. These waves have a sharp crest and wide trough. The solution of this wave theory is in the form of the Jacobi elliptical functions *cn*, and thus this is named as the cnoidal wave theory. Cnoidal wave with infinitely large wavelength and period is known as solitary wave. Solitary waves are caused during hydraulic jumps, tsunami waves and tidal bores. The API RP2A Guidelines provides information on the regions of applicability of these wave theories [7]. The most commonly used wave theories are Airy's and fifth order Stokes' wave theories. Wave theories are used for the determination of the velocity and acceleration of the water particles, which are then incorporated in the Morison equation to generate wave forces acting on thin cylindrical members of offshore structures [8].

A comparative study was conducted by Chandrasekaran et al. [9] on the behaviour of the response of Tension Leg Platform (TLP) caused by regular waves modelled by Airy's and fifth order Stokes' wave theory. It was reported that the surge and pitch responses obtained from Airy's wave theory are greater as compared to those from Stokes' wave theory. A similar investigation for Articulated Loading Platform (ALP) was carried out by Aslam et al. [10]. It was observed that deck displacement, base hinge shear, upper hinge shear, bottom hinge rotation and upper hinge rotation produced by Airy's theory are higher than that produced by Stokes' theory.

Jacket platforms are bottom supported in nature and are the most commonly constructed structures in the marine industry. These are the fixed-type offshore platforms consisting of four to eight legs that are anchored to the seafloor. This type of offshore structure generally supports a superstructure comprising of two to three decks. In this research work, a comparative study is conducted between the wave forces obtained from Morison equation using both Airy's and Stokes' fifth order theories, acting at different levels of two different jacket platforms located at different water depths. The deck displacement responses caused due to these forces are also compared. The study focusses on how unrealistic Airy's wave theory is for high waves in deep water compared to Stokes' fifth order wave theory. Similarly, the study also emphasises on how Stokes' fifth order wave theory results differ from Airy's linear wave theory in shallow water for small wave height.

2. Problem Description

In this study, two different sea states are considered, one for deep water case (d = 120m, H = 8m, T = 8.5sec and another for shallow water case (d = 20m, H = 0.5m, T = 10sec). As regards the applicability of wave theories, the deep water case satisfies Stokes' nonlinear fifth order wave theory while the shallow water case meets Airy's wave theory.

Two jacket platforms are considered, whose designs are very similar to the jacket platform considered by Moharrami and Tootkaboni [11]. The platforms are of height 130m and 30m, situated at a water depth of 120m and 20m respectively. The tubular members have outer diameter equal to 1.5m for leg members and 1m for bracing members. The thickness of the tubular section is 2cm. The platforms are modelled as 8-DOF and 3-DOF lumped mass systems. The elevations of the 130m and 30m jacket platforms with equivalent MDOF system models are shown in Fig. 1 and Fig. 2 respectively. The values of mass and stiffness of the lumped mass models are indicated in Tables 1 and 2. The modal time periods of the two models are listed in Tables 3 and 4.

Table-1. Mass and stiffness values of 130m jacket platform

Level	Parameter	
	Mass, m_i (ton)	Stiffness, k_i (kN/m)
1	305.02	340 x 10^3
2	459.28	290 x 10^3
3	459.28	235 x 10^3
4	459.28	200 x 10^3
5	459.28	175 x 10^3
6	459.28	145 x 10^3
7	292.36	120 x 10^3
8	934.26	50 x 10^3

Table-2. Mass and stiffness values of 30m jacket platform

Level	Parameter	
	Mass, m_i (ton)	Stiffness, k_i (kN/m)
1	305.02	340 x 10^3
2	292.36	120 x 10^3
3	934.26	50 x 10^3

Table-3. Modal periods of 130m jacket platform

Mode Number	Modal Periods (s)
1	1.65
2	0.71
3	0.38
4	0.26
5	0.20
6	0.18
7	0.15
8	0.12

Table-4. Modal periods of 30m jacket platform

Mode Number	Modal Periods (s)
1	1.05
2	0.28
3	0.15

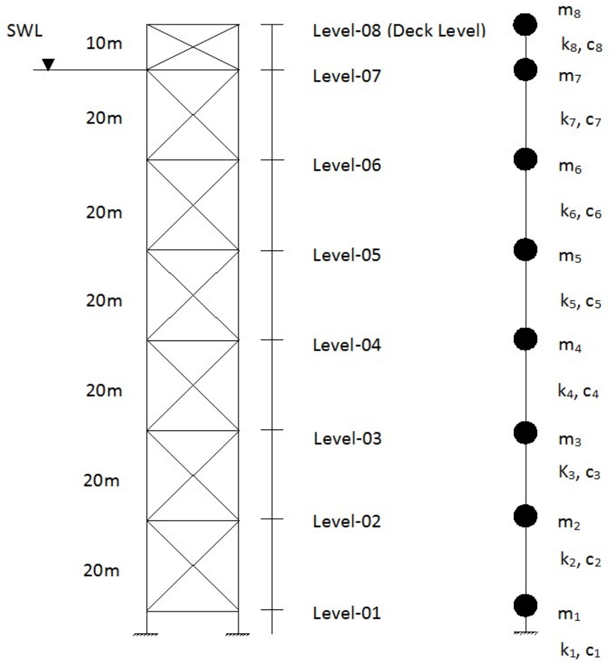

Fig. 1. Elevation of 130m jacket platform with equivalent MDOF system

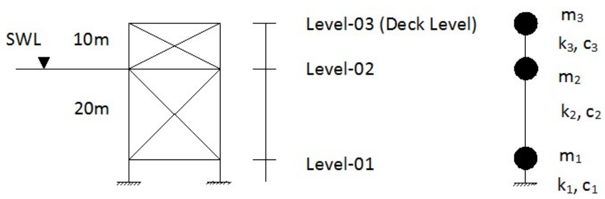

Fig. 2. Elevation of 30m jacket platform with equivalent MDOF system

The equation of motion of the MDOF system model of the jacket platform under a regular wave is provided below,

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F(t)\} \quad (1)$$

where, $[M]$, $[C]$ and $[K]$ represent the mass, damping and stiffness matrices of the jacket platform, respectively. $\{x\} = \{x_1, x_2, \ldots x_i, \ldots x_n\}^T$ represents the displacement vector of jacket platform. $\{\ddot{x}\}$ and $\{\dot{x}\}$ are vectors representing the acceleration and velocity of the jacket structure, respectively. $[C]$ is assumed proportional to $[M]$ and $[K]$ and is computed using Rayleigh damping assuming 2% damping ratio for the first two vibration modes. $\{F(t)\}$ denotes the vector of wave forces acting on jacket platform, comprising both drag and inertia components, as obtained from the Morison equation. Eq. (1) is solved using the fourth-order Runge-Kutta method.

3. Kinematics of Water Particles

Airy's Wave Theory

This wave theory is commonly recognised as the linear wave theory and is the most extensively used wave theory in ocean engineering [1]. This wave is sinusoidal and is applicable where the wave height (H) is less than that of the water depth (d) and the wavelength (λ). Stretching modifications were suggested by Chakrabarti on the linear theory for considering waves above still water level (SWL) [12]. The free surface profile (η) of Airy's wave theory is given by

$$\eta(x,t) = \frac{H}{2}\cos(kx - \omega t) \quad (2)$$

where, k is wave number ($2\pi/\lambda$), and ω is wave frequency ($2\pi/T$).

Components of water particle velocity in the horizontal direction $\dot{u}(x,t)$ and vertical direction $\dot{v}(x,t)$ are provided in Eqs. (3) and (4).

$$\dot{u}(x,t) = \frac{\pi H}{T}\frac{\cosh k(d+z)}{\sinh kd}\cos(kx - \omega t) \quad (3)$$

$$\dot{v}(x,t) = \frac{\pi H}{T}\frac{\cosh k(d+z)}{\sinh kd}\sin(kx - \omega t) \quad (4)$$

Components of water particle acceleration in the horizontal direction $\ddot{u}(x,t)$ and vertical direction $\ddot{v}(x,t)$ are provided in Eqs. (5) and (6).

$$\ddot{u}(x,t) = 2\pi\left(\frac{H}{T}\right)^2\frac{\cosh k(d+z)}{\sinh kd}\sin(kx - \omega t) \quad (5)$$

$$\ddot{v}(x,t) = 2\pi\left(\frac{H}{T}\right)^2\frac{\cosh k(d+z)}{\sinh kd}\cos(kx - \omega t) \quad (6)$$

Stokes' Fifth Order Wave Theory

This wave theory is generally appropriate for higher waves in deep water condition [2]. This wave theory consists of five components. Skjelbreia and Hendrickson [13] suggested an extension of this wave theory. These waves have a steep crest and a shallow trough. The free surface profile (η) of fifth order Stokes' theory is represented by

$$\eta(x,t) = \frac{1}{k}\sum_{n=1}^{5} F_n \cos n(kx - \omega t) \quad (7)$$

Components of water particle velocity in the horizontal direction $\dot{u}(x,t)$ and vertical direction $\dot{v}(x,t)$ are provided in Eqs. (8) and (9).

$$\dot{u}(x,t) = \frac{\omega}{k}\sum_{n=1}^{5} G_n \frac{\cosh(nk(d+z))}{\sinh(nkd)}\cos[n(kx - \omega t)] \quad (8)$$

$$\dot{v}(x,t) = \frac{\omega}{k}\sum_{n=1}^{5} G_n \frac{\sinh(nk(d+z))}{\sinh(nkd)}\sin[n(kx - \omega t)] \quad (9)$$

Components of water particle acceleration in the horizontal direction $\ddot{u}(x,t)$ and vertical direction $\ddot{v}(x,t)$ are provided in Eqs. (10) and (11)

$$\ddot{u}(x,t) = \frac{kc_s^2}{2}\sum_{n=1}^{5} R_n \sin n(kx - \omega t) \quad (10)$$

$$\ddot{v}(x,t) = \frac{-kc_s^2}{2}\sum_{n=1}^{5} S_n \cos n(kx - \omega t) \quad (11)$$

where, F_n, G_n, R_n and S_n are provided in Dawson [1] and Patel [14].

4. Calculation of wave force

The diameter of the leg and bracing members of the jacket platform is much lower than the wavelength of the approaching waves. For such condition, wave forces exerted by approaching waves on the structural members of the platform may be calculated by the Morison equation. The Morison equation calculates wave force from water particles' velocity and acceleration components. The Morison equation is given by

$$F = 0.5C_d \rho D|\dot{u}|\dot{u} + C_m \rho \frac{\pi D^2}{4} \ddot{u} \quad (12)$$

where, ρ represents the density of water, D represents the outer diameter of the cylinder and C_m and C_d are respectively the hydrodynamic inertia and drag coefficients.

5. Numerical Study

A comparison of the wave forces at Levels 1 to 7 of the 130m platform as calculated from Airy's and Stokes' wave theories is presented in Figs. 3-9 respectively. As Level-8 is the deck level, wave force is not present at that level.

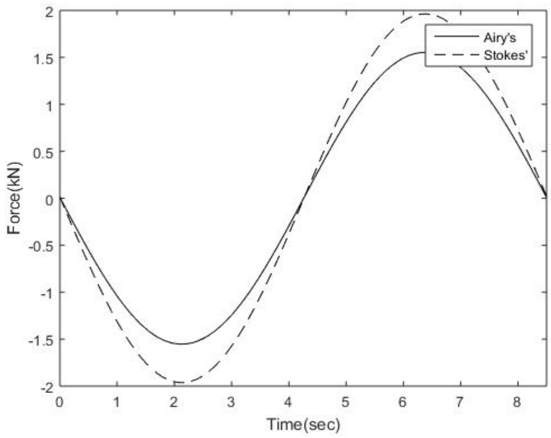

Fig. 3. Wave load acting at Level-01

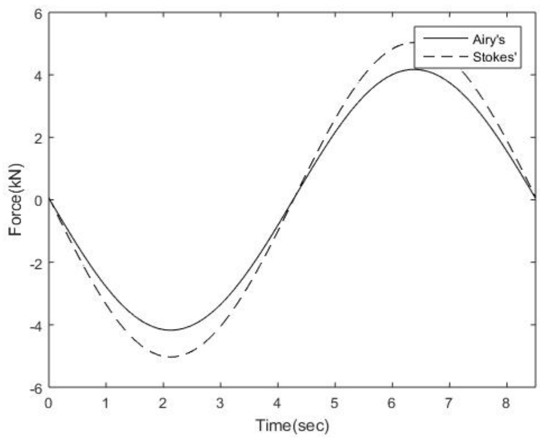

Fig. 4. Wave load acting at Level-02

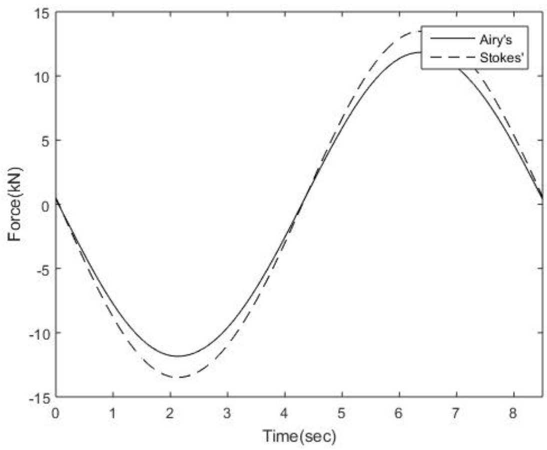

Fig. 5. Wave load acting at Level-03

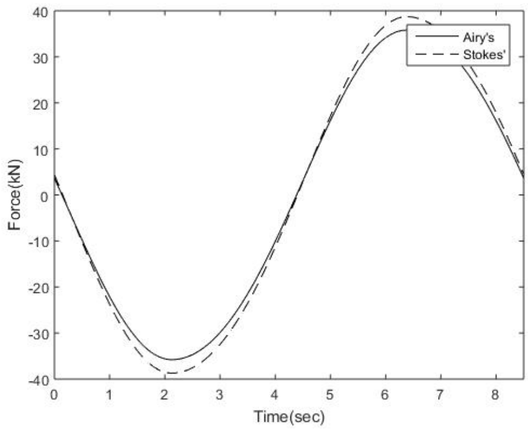

Fig. 6. Wave load acting at Level-04

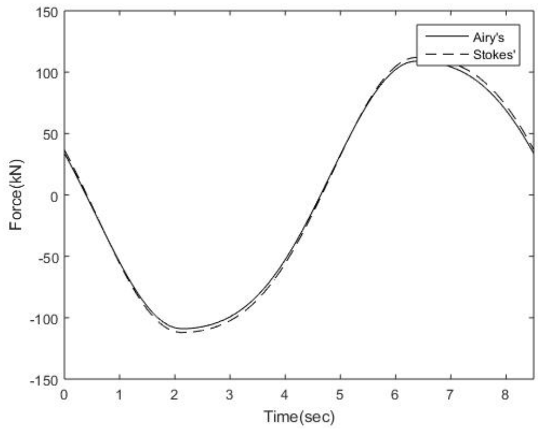

Fig. 7. Wave load acting at Level-05

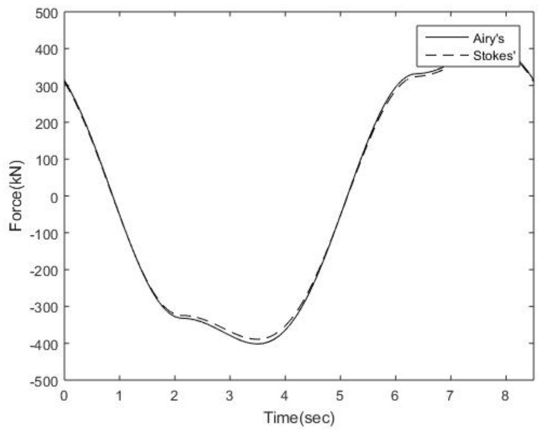

Fig. 8. Wave load acting at Level-06

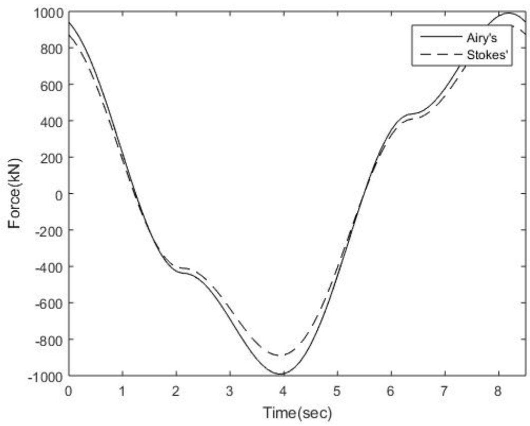

Fig. 9. Wave load acting at Level-07

In case of the deep water condition, it is observed that wave forces acting near SWL on jacket leg and bracing members calculated using Airy’s wave theory are slightly higher compared to the wave forces calculated using fifth order Stokes’ wave theory. However, at increasing depths of water from the SWL, it is found that the wave force magnitudes obtained from the two wave theories are very close and only near the sea bed the values from Airy’s theory reduces from that obtained from Stokes’ fifth order theory. Thus it is observed that, though Airy’s linear wave theory is far more simplified than Stokes’ fifth-order wave theory; there is not much difference in the wave forces predicted by these two wave theories for high waves in deep water. The differences in force values at each level, expressed in percentage of that obtained by Stokes’ wave theory, are provided in Table 5.

A comparison of the wave forces at Levels 1 and 2 of the 30m jacket platform as computed using Airy’s and Stokes’ wave theories is provided in Figs. 10-11 respectively. As Level-3 is the deck level, wave force is not present at that level.

Table-5. Comparison of wave forces on the 130m jacket platform

Level	Maximum Wave Force (kN)		Wave theory producing greater force	Difference (%)
	Airy’s	Stokes’		
1	1.55	1.96	Stokes’	20.92
2	4.17	5.03	Stokes’	17.10
3	11.84	13.49	Stokes’	12.23
4	35.79	38.72	Stokes’	7.57
5	108.96	112.07	Stokes’	2.78
6	401.61	396.04	Airy’s	1.41
7	991.72	922.45	Airy’s	7.51

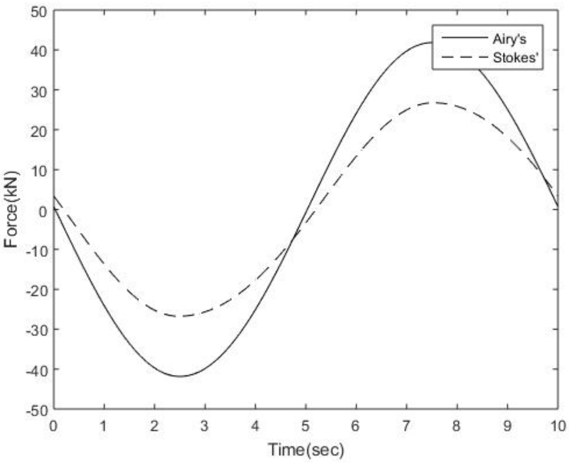

Fig. 10 Wave load acting at Level-01

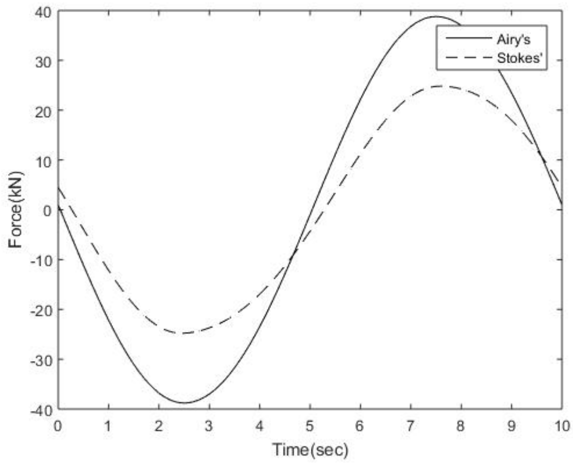

Fig. 11. Wave load acting at Level-02

For the shallow water case, it is observed that wave forces calculated using Stokes’ fifth order wave theory on jacket leg and bracing members at any depth of the sea are significantly lower than those evaluated from Airy’s wave theory. Thus, the study reinforces the knowledge that Stokes’ fifth order wave theory is entirely unsuitable for use in shallow water depth conditions as it grossly underestimates the prediction of the wave forces in these conditions. The differences in force values at each level, expressed as percentage of that obtained by Airy’s wave theory, are provided in Table-6.

The displacement response time histories at the deck level of the 130m jacket platform due to horizontal wave forces calculated using Airy’s and Stokes’ wave theories are presented in Fig. 12. It is observed that the two time histories are very similar.

Table-6. Comparison of wave forces on the 30m jacket platform

Level	Maximum Wave Force (kN)		Wave theory producing greater force	Difference (%)
	Airy’s	Stokes’		
1	41.85	26.76	Airy’s	36.06
2	38.80	24.80	Airy’s	36.08

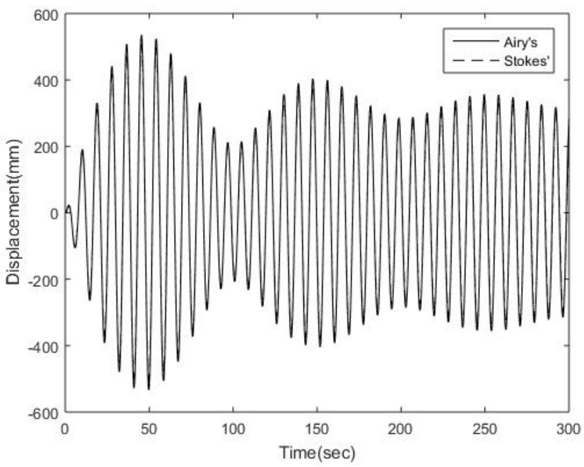

Fig. 12. Deck displacement response of 130m jacket platform

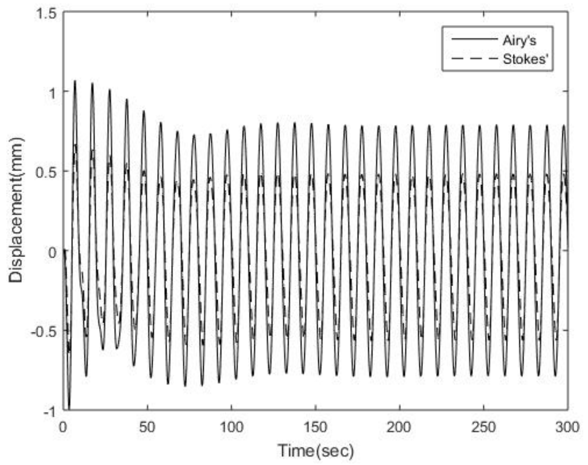

Fig. 13. Deck displacement response of 30m jacket platform

For the 30m jacket platform, the displacement response time histories at deck level due to horizontal wave forces calculated using Airy's and Stokes' wave theories are shown in Fig. 13. The underestimation of the displacement values by the latter is clearly evident from the plot.

The maximum deck displacement in case of the 130m jacket platform due to the horizontal wave forces determined using Airy's and Stokes' wave theories is 535.48mm and 507.21mm respectively, it is 1.07mm and 0.67mm, respectively in case of the 30m jacket platform. It is observed that for while Airy's wave theory provides a reasonably accurate estimation on the conservative side of the peak deck displacement even in deep water conditions, Stokes' wave theory provides inaccurate deck displacement values in shallow water conditions.

6. Conclusions

A comparison of the horizontal wave forces on the leg and bracing members of two example jacket platforms in deep water and in shallow water conditions as obtained considering Airy's and Stokes' wave theories is presented. For the deep water case, the magnitude of the horizontal wave forces on the jacket platform obtained from Airy's wave theory is greater than that from Stokes' wave theory near SWL. Near the seabed it is observed that the horizontal wave force magnitude, though small, computed from Stokes' fifth order theory is more than that obtained from Airy's theory. However, the differences are not substantial. Further, in the deep water case, the deck displacement time histories obtained from the two wave theories are almost indistinguishable. This leads to the inference that the simplified Airy's wave theory may be used for preliminary investigations even for deep water, high wave conditions, though expectedly a more accurate estimation of the wave forces and the deck displacement of the jacket platform necessitates the use of the rigorous Stokes' fifth order wave theory. Unlike the deep water case, in shallow water, the horizontal wave force magnitudes on the jacket platform and the deck displacements calculated from Stokes' fifth order theory are highly unrealistic and inaccurate, the values being much lower than that obtained from Airy's wave theory. This indicates the complete invalidity of Stokes' fifth order theory in shallow water conditions with smaller wave heights, for which it is recommended to use Airy's linear wave theory.

Notation

The various symbols used in the present paper are listed below.

H	=	Wave Height (m)
d	=	Water Depth (m)
T	=	Time period (s)
η	=	Free surface profile
k	=	Wave number
ω	=	Wave frequency (rad/s)
x	=	horizontal coordinate of the point at which η is considered
t	=	Time instant
λ	=	Wavelength (m)
$\dot{u}, \dot{v}$	=	Horizontal and vertical velocity component of water particle velocity respectively (m/s)
$\ddot{u}, \ddot{v}$	=	Horizontal and vertical velocity component of water particle acceleration respectively (m/s^2)
F_n, G_n, R_n, S_n	=	Constants
C_m, C_d	=	Hydrodynamic inertia and drag coefficient respectively
D	=	Diameter of pile (m)
ρ	=	Density of fluid (kg/m^3)

Disclosures

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